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SR-52 MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS--ETC(U)

JAN 79   S W TSAI, H T HAHN, F HUBER

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1 OF 2

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B.S.

# SR-52
# MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS WORKBOOK

*MECHANICS AND SURFACE INTERACTIONS BRANCH*
*NONMETALLIC MATERIALS DIVISION*

JANUARY 1979

TECHNICAL REPORT AFML-TR-77-50

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AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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This technical report has been reviewed and is approved for publication.

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Project Scientist

FOR THE COMMANDER

JMKelble

JEROME M. KELBLE, Chief
Nonmetallic Materials Division

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20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

This is a companion volume to AFML-TR-78-33, "Composite Materials Workbook." Both workbook and its solutions are intended to present to the users of composite materials a set of tools to solve most commonly encountered problems in design and testing. Attempts are made to simplify both the operational and conceptual aspects. Subjects are selected from the standpoint of practical application rather than elegance.

(over)

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20. ABSTRACT (continued)

Programmable pocket calculators, preferably with magnetic card capability, are found to be most suitable to perform the calculations in the workbook. In this report, the description, operating instructions, program listing, and sample problems for these calculations have been compiled for Texas Instruments SR-52.

It is believed that composite materials are simple in concept and easy to describe quantitatively with the aid of card programmable calculators. The performance characteristics of composite materials can now be fully appreciated and utilized.

# FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFML/MBM), Nonmetallic Materials Division, Air Force Materials Laboratory, Wright-Patterson AFB, Ohio. The work was performed under the joint support of Project No. 2419 "Nonmetallic Structural Materials," Task No. 241903 "Composite Materials and Mechanics Technology," and Project No. 2307 "Aerospace Sciences," Task No. 2307P1 "Life Analysis and Failure Mechanics in Engine and Airframe Structural Metals and Composites." The time period covered by the effort was 1 January 1976 to 1 March 1977. Stephen W. Tsai (AFML/MBM) was the laboratory project engineer.

Same numbers for equations and figures are used in this report as those in the AFML-TR-78-33, "Composite Materials Workbook," in order to facilitate cross referencing. Program card numbers are assigned the page number on which the instructions for the program card appear.

The technical assistance from Texas Instruments Incorporated during the preparation of this report is gratefully acknowledged. Standard production cards by Texas Instruments relevant to this report and the Workbook are listed in the Appendix.

## TABLE OF CONTENTS

[redacted]

TABLE OF CONTENTS (continued)

## TABLE OF CONTENTS (concluded)

TITLE: SI/ENGINEERING CONVERSION NO: 001

| | | | | |
|---|---|---|---|---|
| A' $Pa \sim kgf/m^2$ | B' $J \sim ft\ lbf$ | C' $J \sim cal$ | D' $kgm^{-3} \sim lb/yd^3$ | E' $ms^{-1} \sim ft/sec$ |
| A $MPa \sim ksi$ | B $MNm^{-1} \sim kip/in$ | C $N \sim lbf$ | D $kW \sim Hp$ | E $MPam^{\frac{1}{2}} \sim ksi\sqrt{in}$ |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| SI ⇨ Eng<br>Set angular mode switch to D (degree) | 1 | 001 A& B | | | |
| | 2 | $Q_{ij}$ $\sigma_i$ | kPa | A | psi |
| | | | MPa | A | ksi |
| | | | GPa | A | $10^6$ psi |
| | | | Pa | A' | $kgf/m^2$ |
| | | | MPa | A' | $kgf/mm^2$ |
| | | $A_{ij}$, $N_i$ | $MNm^{-1}$ | B | kip/in |
| | | | $Nm^{-1}$ | A' | kgf/m |
| | | $B_{ij}$, $M_i$ | N | C | lbf |
| | | | kN | C | kip |
| | | | N | A' | kgf |
| | | $D_{ij}$, J | Nm | B' | ft lbf |
| | | | Nm | A' | kgf/m |
| SI ⇦ Eng<br>Set angular mode switch to R (radian) | 3 | $Q_{ij}$, $\sigma_i$ | psi | A | kPa |
| | | | ksi | A | MPa |
| | | | $10^6$ psi | A | GPa |
| | | | $kgf/m^2$ | A' | Pa |
| | | | $kgf/mm^2$ | A' | MPa |
| | | $A_{ij}$, $N_i$ | kip/in | B | $MNm^{-1}$ |
| | | | kgf/m | A' | $Nm^{-1}$ |
| | | $B_{ij}$, $M_i$ | lbf | C | N |
| | | | kip | C | kN |
| | | | kgf | A' | N |
| | | $D_{ij}$, J | ft lbf | B' | Nm |
| | | | kgf m | A' | Nm |

**PROGRAM TITLE:** SI/ENGINEERING CONVERSION **NO:** 001

**PROGRAM LISTING:**

```
000 46  045 51  090 03  135 95
001 11  046 89  091 07  136 81
002 42  047 01  092 06  137 46
003 00  048 93  093 95  138 89
004 00  049 03  094 81  139 59
005 51  050 04  095 46  140 32
006 89  051 01  096 18  141 90
007 93  052 95  097 42  142 87
008 01  053 81  098 00  143 43
009 04  054 46  099 00  144 00
010 05  055 15  100 51  145 00
011 95  056 42  101 89  146 65
012 81  057 00  102 93  147 56
013 46  058 00  103 02  148 46
014 12  059 51  104 03  149 87
015 42  060 89  105 08  150 43
016 00  061 93  106 09  151 00
017 00  062 09  107 95  152 00
018 51  063 00  108 81  153 55
019 89  064 09  109 46  154 56
020 05  065 08  110 19
021 93  066 95  111 42
022 07  067 81  112 00
023 01  068 46  113 00
024 95  069 16  114 51
025 81  070 42  115 89
026 46  071 00  116 01
027 13  072 00  117 93
028 42  073 51  118 06
029 00  074 89  119 08
030 00  075 93  120 06
031 51  076 01  121 95
032 89  077 00  122 81
033 93  078 02  123 46
034 02  079 95  124 10
035 02  080 81  125 42
036 04  081 46  126 00
037 08  082 17  127 00
038 95  083 42  128 51
039 81  084 00  129 89
040 46  085 00  130 03
041 14  086 51  131 93
042 42  087 89  132 02
043 00  088 93  133 08
044 00  089 07  134 01
045 51  090 03  135 95
```

TITLE: VOID CONTENT

NO: 003

| | | | | |
|---|---|---|---|---|
| A' $v_v$ | B' $v_f{}'$ | C' $v_f$ | D' | E' |
| A $\rho_f$ | B $\rho_m$ | C $m_f$ | D $m_m$ | E $\rho$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $\rho_f$ | 02 $\rho_m$ | 03 $m_f$ | 04 $m_m$ | 05 $\rho$ | 06 $v_v$ |
| 07 $v_f'$ | 08 | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input Data | 1 | 003 A&B | $\rho_f$ | A | $\rho_f$ |
| | | | $\rho_m$ | B | $\rho_m$ |
| | | | $m_f$ | C | $m_f$ |
| | | | $m_m$ | D | $m_m$ |
| | | | $\rho$ | E | $\rho$ |
| Compute | 2 | | | A' | $v_v$ |
| | | | | B' | $v_f{}'$ |
| | | | | C' | $v_f$ |

PROGRAM TITLE: VOID CONTENT NO: 003

FORMULAS:

$m_{f,m}$ : mass fraction of fiber, matrix

$\rho_{f,m}$ : density of fiber, matrix

a. Void content

$$v_v = 1 - \rho (m_f/\rho_f + m_m/\rho_m)$$

b. Apparent fiber volume fraction ($v_v = 0$)

$$v_f' = \frac{1}{1 + \frac{m_m}{m_f} \frac{\rho_f}{\rho_m}}$$

c. True fiber volume fraction

$$v_f = (1 - v_v)v_f'$$

**PROGRAM TITLE:** VOID CONTENT **NO:** 003

**PROGRAM LISTING:**

```
000 46   045 01   090 53
001 11   046 85   091 01
002 42   047 43   092 75
003 00   048 00   093 43
004 01   049 04   094 00
005 81   050 55   095 06
006 46   051 43   096 54
007 12   052 00   097 65
008 42   053 02   098 43
009 00   054 54   099 00
010 02   055 95   100 07
011 81   056 42   101 95
012 46   057 00   102 81
013 13   058 06
014 42   059 81
015 00   060 46
016 03   061 17
017 81   062 01
018 46   063 55
019 14   064 53
020 42   065 01
021 00   066 85
022 04   067 43
023 81   068 00
024 46   069 04
025 15   070 65
026 42   071 43
027 00   072 00
028 05   073 01
029 81   074 55
030 46   075 43
031 16   076 00
032 01   077 03
033 75   078 55
034 43   079 43
035 00   080 00
036 05   081 02
037 65   082 54
038 53   083 95
039 43   084 42
040 00   085 00
041 03   086 07
042 55   087 81
043 43   088 46
044 00   089 18
```

SAMPLE PROBLEM: 003 VOID CONTENT NO: 003

1) Input: $\rho_f = 2600\ \text{Kg-m}^{-3}$

$\rho_m = 1200\ \text{Kg-m}^{-3}$

$m_f = .65$

$m_m = .35$

$\rho = 1800\ \text{Kg-m}^{-3}$

Output: $V_v = .025$

$V_f' = .462$

$V_f = .450$

2) Input: $\rho_f = 1700\ \text{Kg-m}^{-3}$

$\rho_m = 1200\ \text{Kg-m}^{-3}$

$m_f = .70$

$m_m = .30$

$\rho = 1500\ \text{Kg-m}^{-3}$

Output: $V_v = .007$

$V_f' = .622$

$V_f = .618$

TITLE: STRESS COMPONENTS NO: 007

| | | | | |
|---|---|---|---|---|
| A' $\sigma_i$ | B' | C' | D' | E' |
| A Initialize | B $\sigma_1'$ | C $\sigma_2'$ | D $\sigma_6'$ | E $\delta$ |

REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $2(\theta-\delta)$ | | | | | |
| 01 | 02 | 03 | 04 | 05 | 06 |
| 07 $\sigma_1'$, $\sigma_1$ | 08 $\sigma_2'$, $\sigma_2$ | 09 $\sigma_6'$, $\sigma_6$ | 10 $(\sigma_1-\sigma_2)/2$ | | |
| 11 | 12 | 13 | 14 | 15 I | 16 R |
| 17 | 18 | 19 | | | |
| 98 | 99 $\delta$ | | | | |

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input $\sigma_i$ | 1 | 007 A&B | $\sigma_1$, $\sigma_2$<br>$\sigma_6$ | A'<br>RUN<br>RUN | $\sigma_1$, $\sigma_2$<br>$\sigma_6$ |
| Compute Invariants | 2 | | | A | $\delta$* |
| Input θ | 3 | | θ(deg) | B | $\sigma_1'$ |
| | | | | C | $\sigma_2'$ |
| | | | | D | $\sigma_6'$ |
| | | | | E | $\delta$ |
| Principal Stresses | 4 | | δ(deg) | B | $\sigma_I$ |
| | | | | C | $\sigma_{II}$ |
| | | | | D | 0 |
| Max Shear | 5 | | δ + 45° | B | $\sigma_1' = I$ |
| | | | | C | $\sigma_2' = I$ |
| | | | | D | $\sigma_6' = R$ |

*Note: Display will flash "0" when $\sigma_1 = \sigma_2$ (See Eq. 30). Stabilize display with CE and proceed, computations are not effected.

**PROGRAM TITLE:** STRESS COMPONENTS **NO:** 007

**FORMULAS:**

TABLE 3 STRESS TRANSFORMATION RELATIONS

|  | I | R |
|---|---|---|
| $\sigma'_x$ | 1 | $\cos 2(\theta - \delta)$ |
| $\sigma'_y$ | 1 | $-\cos 2(\theta - \delta)$ |
| $\sigma'_{xy}$ | 0 | $-\sin 2(\theta - \delta)$ |

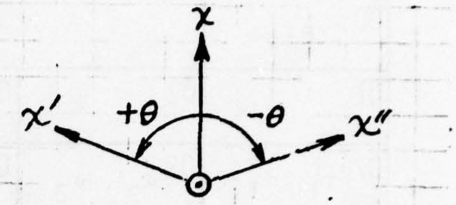


Figure 12 New coordinates x'- y' in terms of old coordinates x-y, or given x-y and θ, find x'-y'. Arrow of rotation is pointing up. If θ is negative, new coordinates are x''-y''.

$$I = \frac{\sigma_x + \sigma_y}{2} \tag{28}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_{xy}^2} \tag{29}$$

$$\tan 2\delta_1 = \frac{2\sigma_{xy}}{\sigma_x - \sigma_y} \tag{30}$$

$\delta = \delta_1 + 90$ if $\sigma_x < \sigma_y$; $\delta = \delta_1$ if $\sigma_x > \sigma_y$

Both I and R are invariants, as shown in the Mohr's Circle.

Special orientations:

(1) When $\theta = \delta$, i.e., Principal Directions

$\sigma'_x$, $\sigma'_y$ = maximum or minimum

$\sigma'_{xy} = 0$

(2) When $\theta - \delta = \pm \pi/4$, i.e., max. shear orientation

$\sigma'_x = \sigma'_y = I$

$\sigma'_{xy}$ = max. = R

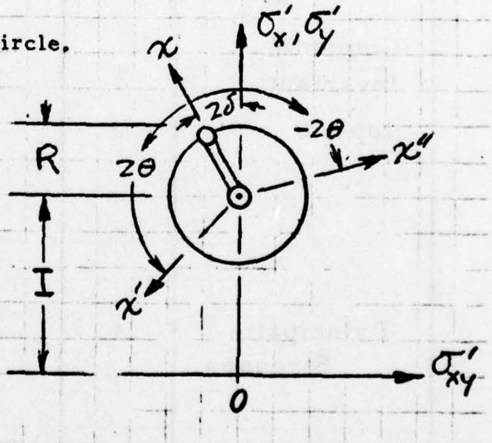


Figure 13 Mohr's Circle is defined by invariants I and R. Phase angle δ is defined by a specific combination of stress components $\sigma_x$, $\sigma_y$ and $\sigma_{xy}$. As reference coordinates change by +θ, the rotation in Mohr's Circle is 2θ.

**PROGRAM TITLE:** STRESS COMPONENTS **NO:** 007

**PROGRAM LISTING:**

```
000 46   045 85   090 09   135 01
001 16   046 43   091 09   136 06
002 42   047 00   092 81   137 95
003 00   048 09   093 46   138 42
004 07   049 40   094 12   139 00
005 81   050 95   095 75   140 08
006 42   051 30   096 43   141 81
007 00   052 42   097 09   142 46
008 08   053 01   098 09   143 14
009 81   054 06   099 95   144 43
010 42   055 43   100 65   145 00
011 00   056 00   101 02   146 00
012 09   057 09   102 95   147 32
013 81   058 55   103 42   148 94
014 46   059 43   104 00   149 65
015 11   060 01   105 00   150 43
016 43   061 00   106 43   151 01
017 00   062 95   107 01   152 06
018 07   063 22   108 05   153 95
019 85   064 34   109 85   154 42
020 43   065 55   110 43   155 00
021 00   066 02   111 00   156 09
022 08   067 95   112 00   157 81
023 95   068 42   113 33   158 46
024 55   069 09   114 65   159 15
025 02   070 09   115 43   160 43
026 95   071 43   116 01   161 09
027 42   072 01   117 06   162 09
028 01   073 00   118 95   163 81
029 05   074 80   119 42
030 43   075 87   120 00
031 00   076 43   121 07
032 07   077 09   122 81
033 75   078 09   123 46
034 43   079 85   124 13
035 00   080 09   125 43
036 08   081 00   126 01
037 95   082 95   127 05
038 55   083 42   128 75
039 02   084 09   129 43
040 95   085 09   130 00
041 42   086 81   131 00
042 01   087 46   132 33
043 00   088 87   133 65
044 40   089 43   134 43
```

## SAMPLE PROBLEM: 007 STRESS COMPONENTS NO: 007

1. Given $\sigma_i = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ (MPa)

$I = 1.5$  $R = -3.041$  $\delta = 49.731$

| | $\delta-45$ | $\delta-30$ | $\delta-15$ | $\delta$ | $\delta+15$ | $\delta+30$ | $\delta+45$ | $\delta-\delta=0$ |
|---|---|---|---|---|---|---|---|---|
| $\sigma'_x$ | 1.5 | 3.021 | 4.134 | 4.541 | 4.134 | 3.021 | 1.5 | 1 |
| $\sigma'_y$ | 1.5 | -.021 | -1.134 | -1.541 | -1.134 | -.021 | 1.5 | 2 |
| $\sigma'_{xy}$ | 3.041 | 2.634 | 1.521 | 0 | -1.521 | -2.634 | -3.041 | 3 |

2. Given $\sigma_i = \begin{Bmatrix} -1 \\ -2 \\ -3 \end{Bmatrix}$ (MPa)

$I = -1.5$  $R = -3.041$  $\delta = -40.269$

| | $\delta-45$ | $\delta-30$ | $\delta-15$ | $\delta$ | $\delta+15$ | $\delta+30$ | $\delta+45$ | $\delta-\delta=0$ |
|---|---|---|---|---|---|---|---|---|
| $\sigma'_x$ | -1.5 | .021 | 1.134 | 1.541 | 1.134 | .021 | -1.5 | -1 |
| $\sigma'_y$ | -1.5 | -3.021 | -4.134 | -4.541 | -4.134 | -3.021 | -1.5 | -2 |
| $\sigma'_{xy}$ | 3.041 | 2.634 | 1.521 | .0 | -1.521 | -2.634 | -3.041 | -3 |

| TITLE: STRAIN COMPONENTS | | | | NO: 011 |
|---|---|---|---|---|
| A' $e_i$ | B' | C' | D' | E' |
| A Initialize | B $e_1'$ | C $e_2'$ | D 6 $e_6'$ | E δ |

| 00 $2(\theta-\delta)$ | | | | | |
|---|---|---|---|---|---|

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 01 | 02 | 03 | 04 | 05 I | 06 R |
| 07 | 08 | 09 | 10 $(e_1-e_2)/2$ | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 $e_1'$, $e_1$ | 18 $e_2'$, $e_2$ | 19 $e'$, $e_6$ | | | |
| 98 δ | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input $e_i$ | 1 | 011 A&B | $e_1$ | A' | $e_1$ |
| | | | $e_2$ | RUN | $e_2$ |
| | | | $e_6$ | RUN | $e_6$ |
| Compute I's Invariants | 2 | | | A | δ* |
| Input θ | 3 | | θ (deg) | B | $e_1'$ |
| Recall | 3' | | | C | $e_2'$ |
| | | | | D | $e_6'$ |
| | | | | E | δ |
| Principal Strains | 4 | | δ (deg) | B | $e_I$ |
| | | | | C | $e_{II}$ |
| | | | | D | 0 |
| Max Shear | 5 | | δ + 45° | B | $e_1'$ = I |
| | | | | C | $e_2'$ = I |
| | | | | D | $e_6'$ = 2R |

*Note: Display will flash "0" when $e_x = e_y$ (See Eq. 44). Stabilize display with CE and proceed, computations are not effected.

**PROGRAM TITLE:** STRAIN COMPONENTS **NO:** 011

**FORMULAS:**

**Only the shear component is the difference between the strain and stress transformation relations, i.e.,**

TABLE 7 STRAIN TRANSFORMATION RELATIONS

| | I | R |
|---|---|---|
| $e'_x$ | 1 | $\cos 2(\theta - \delta)$ |
| $e'_y$ | 1 | $-\cos 2(\theta - \delta)$ |
| $e'_{xy}$ | 0 | $-2\sin 2(\theta - \delta)$ |

$$I = \frac{e_x + e_y}{2} \qquad (42)$$

$$R = \sqrt{\left(\frac{e_x - e_y}{2}\right)^2 + \left(\frac{e_{xy}}{2}\right)^2} \qquad (43)$$

$$\tan 2\delta = \frac{e_{xy}}{e_x - e_y} \qquad (44)$$

$\delta = \delta_1$ if $e_x > e_y$

$\delta = \delta_1 + 90$ if $e_x < e_y$

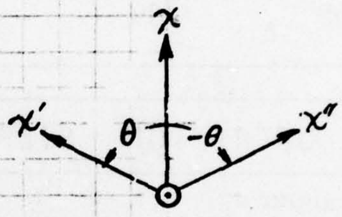


**Figure 21 Positive and negative rotations for transformation.**

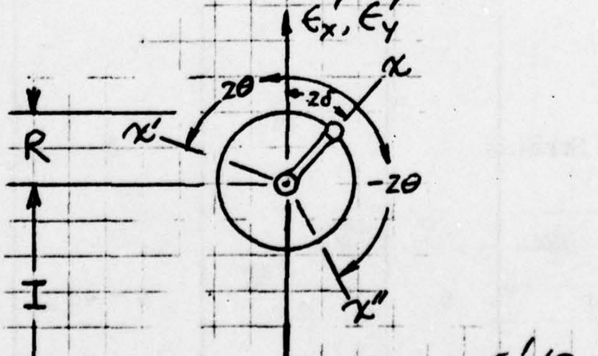


**Figure 22 Mohr's Circle representation of strain transformation. Only the one-half factor for the shear strain is the difference between the strain representation and that for stress.**

**PROGRAM TITLE:** STRAIN COMPONENTS **NO:** 011

**PROGRAM LISTING:**

```
000 46   045 08   090 00   135 00
001 16   046 95   091 95   136 05
002 42   047 55   092 42   137 75
003 01   048 02   093 09   138 43
004 07   049 95   094 08   139 00
005 81   050 42   095 81   140 00
006 42   051 01   096 46   141 33
007 01   052 00   097 87   142 65
008 08   053 40   098 43   143 43
009 81   054 85   099 09   144 00
010 42   055 43   100 08   145 06
011 01   056 01   101 81   146 95
012 09   057 09   102 46   147 42
013 81   058 40   103 12   148 01
014 46   059 95   104 75   149 08
015 11   060 30   105 43   150 81
016 43   061 42   106 09   151 46
017 01   062 00   107 08   152 14
018 08   063 06   108 95   153 43
019 85   064 43   109 65   154 00
020 43   065 01   110 02   155 00
021 01   066 09   111 95   156 32
022 07   067 55   112 42   157 94
023 95   068 43   113 00   158 65
024 55   069 01   114 00   159 02
025 02   070 00   115 43   160 65
026 95   071 95   116 00   161 43
027 42   072 22   117 05   162 00
028 00   073 34   118 85   163 06
029 05   074 55   119 43   164 95
030 43   075 02   120 00   165 42
031 01   076 95   121 00   166 01
032 09   077 42   122 33   167 09
033 55   078 09   123 65   168 81
034 02   079 08   124 43   169 46
035 95   080 43   125 00   170 15
036 42   081 01   126 06   171 43
037 01   082 00   127 95   172 09
038 09   083 80   128 42   173 08
039 43   084 87   129 01   174 81
040 01   085 43   130 07
041 07   086 09   131 81
042 75   087 08   132 46
043 43   088 85   133 13
044 01   089 09   134 43
```

SAMPLE PROBLEM: 011 STAIN COMPONENTS NO: 011

1. Given $e_i = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ (mm/m or $10^3 \mu m/m$)

$I = 1.5$ $R = -1.581$ $\delta = 54.217$

| | $\delta-45$ | $\delta-30$ | $\delta-15$ | $\delta$ | $\delta+15$ | $\delta+30$ | $\delta+45$ | $\delta-\delta=0$ |
|---|---|---|---|---|---|---|---|---|
| $e'_x$ | 1.5 | 2.291 | 2.869 | 3.081 | 2.869 | 2.291 | 1.5 | 1 |
| $e'_y$ | 1.5 | .709 | .131 | -.081 | .131 | .709 | 1.5 | 2 |
| $e'_{xy}$ | 3.162 | 2.739 | 1.581 | 0 | -1.581 | -2.739 | -3.162 | 3 |

2. Given $e_i = \begin{Bmatrix} -1 \\ -2 \\ -3 \end{Bmatrix}$ (mm/m or $10^3 \mu m/m$)

$I = -1.5$ $R = -1.581$ $\delta = -35.783$

| | $\delta-45$ | $\delta-30$ | $\delta-15$ | $\delta$ | $\delta+15$ | $\delta+30$ | $\delta+45$ | $\delta-\delta=0$ |
|---|---|---|---|---|---|---|---|---|
| $e'_x$ | -1.50 | - .709 | - .131 | .081 | - .131 | - .709 | -1.50 | -1 |
| $e'_y$ | -1.50 | -2.291 | -2.869 | -3.081 | -2.869 | -2.291 | -1.50 | -2 |
| $e'_{xy}$ | 3.162 | 2.739 | 1.581 | 0 | -1.581 | -2.739 | -3.162 | -3 |

TITLE: STRAIN ROSETTES NO: 015

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $\theta_i$ | B $e_i$ | C Initialize | D | E Solve 3x3 |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $\cos^2 A$ | | | | | |
| 01 $\sin^2 A$ | 02 sinAcosA | 03 $e_A$ | 04 z | 05 $\cos^2 B$ | 06 $\sin^2 B$ |
| 07 sinBcosB | 08 $e_B$ | 09 y | 10 $\cos^2 C$ | | |
| 11 $\sin^2 C$ | 12 sinCcosC | 13 $e_C$ | 14 $\theta_A$ | 15 $\theta_B$ | 16 $\theta_C$ |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Compute Coeff | 1 | 015 A& B | | C | |
| Input Angles | 2 | | $\theta_A$(deg) | A | $\theta_A$ |
| | | | $\theta_B$(deg) | RUN | $\theta_B$ |
| | | | $\theta_C$(deg) | RUN | $\theta_C$ |
| Input Strains | 3 | | $e_A$ | B | $e_A$ |
| | | | $e_B$ | RUN | $e_B$ |
| | | | $e_C$ | RUN | $e_C$ |
| Solve 3 x 3 | 4 | MA1-05 | | E | $e_x$ |
| | | | | RUN | $e_y$ |
| | | | | RUN | $e_{xy}$ |

## FORMULAS:

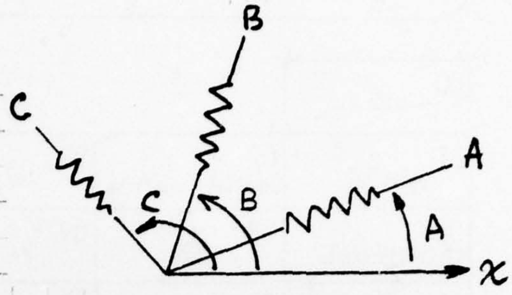


Figure 25 Strain rosette orientation with positive angles. If an angle is negative, it shall be so entered into Equation and

**a. Three-Element Rosettes**

Since there are 3 strain components at each point, 3-element rosettes are in general needed to solve for 3 unknowns. Assuming 3 elements are mounted at 3 different angles from some reference coordinates, each rosette must satisfy the first equation in Eq. 45, i.e.

$$\begin{aligned} e_A &= (\cos^2 A)e_x + (\sin^2 A)e_y + (\sin A \cos A)e_{xy} \\ e_B &= (\cos^2 B)e_x + (\sin^2 A)e_y + (\sin B \cos B)e_{xy} \qquad (45) \\ e_C &= (\cos^2 C)e_x + (\sin^2 C)e_y + (\sin C \cos C)e_{xy} \end{aligned}$$

In matrix form

$$\begin{bmatrix} \cos^2 A & \sin^2 A & \sin A \cos A \\ \cos^2 B & \sin^2 B & \sin B \cos B \\ \cos^2 C & \sin^2 C & \sin C \cos C \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} = \begin{Bmatrix} e_A \\ e_B \\ e_C \end{Bmatrix} \qquad (46)$$

PROGRAM TITLE: STRAIN ROSETTES NO: 015

PROGRAM LISTING:

```
000 46   045 01   090 06
001 11   046 43   091 33
002 42   047 01   092 40
003 01   048 04   093 42
004 04   049 33   094 01
005 81   050 65   095 00
006 42   051 43   096 43
007 01   052 01   097 01
008 05   053 04   098 06
009 81   054 32   099 32
010 42   055 95   100 40
011 01   056 42   101 42
012 06   057 00   102 01
013 81   058 02   103 01
014 46   059 43   104 43
015 12   060 01   105 01
016 42   061 05   106 06
017 00   062 33   107 33
018 03   063 40   108 65
019 81   064 42   109 43
020 42   065 00   110 01
021 00   066 05   111 06
022 08   067 43   112 32
023 81   068 01   113 95
024 42   069 05   114 42
025 01   070 32   115 01
026 03   071 40   116 02
027 81   072 42   117 81
028 46   073 00
029 13   074 06
030 43   075 43
031 01   076 01
032 04   077 05
033 33   078 33
034 40   079 65
035 42   080 43
036 00   081 01
037 00   082 05
038 43   083 32
039 01   084 95
040 04   085 42
041 32   086 00
042 40   087 07
043 42   088 43
044 00   089 01
```

SAMPLE PROBLEM: 015 STRAIN ROSETTES NO: 015

1) Given A=0, B=60°, C=-60° and $\epsilon_A$, $\epsilon_B$, $\epsilon_C$ = 10, -3, -3

Find Poisson's ratio along a-axis

$\epsilon_x$ = 10

$\epsilon_y$ = -7.333

$\epsilon_{xy}$ = $-4.618 \times 10^{-12} \simeq 0$

$\nu$ = .733

TITLE: STRESS-STRAIN RELATION NO: 019

| | | | | |
|---|---|---|---|---|
| A' $S_{ij}$ | B' | C' | D' | E' |
| A $\sigma_i$ | B $e_1$ | C $e_2$ | D $e_6$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $S_{11}$ | 02 $S_{22}$ | 03 $S_{12}$ | 04 $S_{66}$ | 05 $S_{16}$ | 06 $S_{26}$ |
| 07 $\sigma_1$ | 08 $\sigma_2$ | 09 $\sigma_6$ | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 $e_1$ | 18 $e_2$ | 19 $e_6$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input $S_{ij}$ | 1 | 019 A&B | $S_{11}$ | A' | $S_{11}$ |
| | | | $S_{22}$ | RUN | $S_{22}$ |
| | | | $S_{12}$ | RUN | $S_{12}$ |
| | | | $S_{66}$ | RUN | $S_{66}$ |
| | | | $S_{16}$ | RUN | $S_{16}$ |
| | | | $S_{26}$ | RUN | $S_{26}$ |
| Input $\sigma_i$ | 2 | | $\sigma_1$ | A | $\sigma_1$ |
| | | | $\sigma_2$ | RUN | $\sigma_2$ |
| | | | $\sigma_6$ | RUN | $\sigma_6$ |
| Strains | 3 | | | B | $e_1$ |
| | | | | C | $e_2$ |
| | | | | D | $e_6$ |
| Transf. Strains | 4 | 011 A&B | | A | $\delta^*$ |
| | | | $\theta$ (deg) | B | $e_1'$ |
| | | | | C | $e_2'$ |
| | | | | D | $e_6'$ |

PROGRAM TITLE: STRESS-STRAIN RELATION NO: 019

FORMULAS:

$$\begin{aligned} \epsilon_1 &= S_{1j}\sigma_j = S_{11}\sigma_1 + S_{12}\sigma_2 + S_{16}\sigma_6 \\ \epsilon_2 &= S_{2j}\sigma_j = S_{21}\sigma_1 + S_{22}\sigma_2 + S_{26}\sigma_6 \\ &\cdot \\ \epsilon_6 &= S_{6j}\sigma_j = S_{61}\sigma_1 + S_{62}\sigma_2 + S_{66}\sigma_6 \end{aligned} \tag{49}$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{21} & S_{22} & S_{26} \\ S_{61} & S_{62} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} \tag{52}$$

$S_{ij}$ is symmetric, i.e., $S_{ij} = S_{ji}$, or $S_{12} = S_{21}$, $S_{16} = S_{61}$, $S_{26} = S_{62}$

**PROGRAM TITLE:** STRESS-STRAIN RELATION **NO:** 019

**PROGRAM LISTING:**

```
000 46   045 65   090 06
001 16   046 43   091 65
002 42   047 00   092 43
003 00   048 07   093 00
004 01   049 85   094 09
005 81   050 43   095 95
006 42   051 00   096 42
007 00   052 03   097 01
008 02   053 65   098 08
009 81   054 43   099 81
010 42   055 00   100 46
011 00   056 08   101 14
012 03   057 85   102 43
013 81   058 43   103 00
014 42   059 00   104 05
015 00   060 05   105 65
016 04   061 65   106 43
017 81   062 43   107 00
018 42   063 00   108 07
019 00   064 09   109 85
020 05   065 95   110 43
021 81   066 42   111 00
022 42   067 01   112 06
023 00   068 07   113 65
024 06   069 81   114 43
025 81   070 46   115 00
026 46   071 13   116 08
027 11   072 43   117 85
028 42   073 00   118 43
029 00   074 03   119 00
030 07   075 65   120 04
031 81   076 43   121 65
032 42   077 00   122 43
033 00   078 07   123 00
034 08   079 85   124 09
035 81   080 43   125 95
036 42   081 00   126 42
037 00   082 02   127 01
038 09   083 65   128 09
039 81   084 43   129 81
040 46   085 00
041 12   086 08
042 43   087 85
043 00   088 43
044 01   089 00
```

1. Given:

$$\sigma_i = \begin{Bmatrix} 400 \\ 60 \\ 15 \end{Bmatrix} (\mathrm{MPa}) = \begin{Bmatrix} .40 \\ .06 \\ .015 \end{Bmatrix} (\mathrm{TPa})$$

$$S_{ij} = \begin{bmatrix} 5.5249 & -1.5470 & 0 \\ -1.5470 & 97.0874 & 0 \\ 0 & 0 & 139.4700 \end{bmatrix} (\mathrm{TPa})^{-1}$$

(for T-300/5208)

$$\epsilon_i = \begin{Bmatrix} 2.1171 \\ 5.2064 \\ 2.0921 \end{Bmatrix} (\mathrm{mm/m})$$

2. Given:

$$\sigma_i = \begin{Bmatrix} .01 \\ .02 \\ .03 \end{Bmatrix} (\mathrm{TPa})$$

$S_{ij}$ $(\mathrm{TPa})^{-1}$ for T-300/5208

$$\epsilon_i = \begin{Bmatrix} .0243 \\ 1.9263 \\ 4.1841 \end{Bmatrix} (\mathrm{mm/m}).$$

TITLE: STRAIN-STRESS RELATION NO: 023

| | | | | |
|---|---|---|---|---|
| A' $Q_{ij}$ | B' | C' | D' | E' |
| A $e_i$ | B $\sigma_1$ | C $\sigma_2$ | D $\sigma_6$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 | 02 | 03 | 04 | 05 | 06 |
| 07 $\sigma_1$ | 08 $\sigma_2$ | 09 $\sigma_6$ | 10 | | |
| 11 $Q_{11}$ | 12 $Q_{22}$ | 13 $Q_{12}$ | 14 $Q_{66}$ | 15 $Q_{16}$ | 16 $Q_{26}$ |
| 17 $e_1$ | 18 $e_2$ | 19 $e_6$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input $Q_{ij}$ | 1 | 023 A&B | $Q_{11}$ | A' | $Q_{11}$ |
| | | | $Q_{22}$ | RUN | $Q_{22}$ |
| | | | $Q_{12}$ | RUN | $Q_{12}$ |
| | | | $Q_{66}$ | RUN | $Q_{66}$ |
| | | | $Q_{16}$ | RUN | $Q_{16}$ |
| | | | $Q_{26}$ | RUN | $Q_{26}$ |
| Input $e_i$ | 2 | | $e_1$ | A | $e_1$ |
| | | | $e_2$ | RUN | $e_2$ |
| | | | $e_6$ | RUN | $e_6$ |
| Stresses | 3 | | | B | $\sigma_1$ |
| | | | | C | $\sigma_2$ |
| | | | | D | $\sigma_6$ |
| Transf. Stresses | 4 | 011 A&B | | A | $\delta^*$ |
| | | | $\theta$ (deg) | B | $\sigma_1'$ |
| | | | | C | $\sigma_2'$ |
| | | | | D | $\sigma_6'$ |

**PROGRAM TITLE:** STRAIN-STRESS RELATION **NO:** 023

**FORMULAS:**

$$\sigma_i = Q_{ij} e_j \tag{50}$$

$$\sigma_1 = Q_{1i} e_i = Q_{11} e_1 + Q_{12} e_2 + Q_{16} e_6$$

$$\sigma_2 = Q_{2i} e_i = Q_{21} e_1 + Q_{22} e_2 + Q_{26} e_6 \tag{51}$$

$$\sigma_6 = Q_{6i} e_i = Q_{61} e_1 + Q_{62} e_2 + Q_{66} e_6$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ e_6 \end{Bmatrix} \tag{53}$$

$Q_{ij}$ is also symmetric, i.e., $Q_{ij} = Q_{ji}$, or $Q_{12} = Q_{21}$, $Q_{16} = Q_{61}$, $Q_{26} = Q_{62}$

**PROGRAM TITLE:** STRAIN-STRESS RELATION **NO:** 023

**PROGRAM LISTING:**

```
000 46   045 65   090 06
001 16   046 43   091 65
002 42   047 01   092 43
003 01   048 07   093 01
004 01   049 85   094 09
005 81   050 43   095 95
006 42   051 01   096 42
007 01   052 03   097 00
008 02   053 65   098 08
009 81   054 43   099 81
010 42   055 01   100 46
011 01   056 08   101 14
012 03   057 85   102 43
013 81   058 43   103 01
014 42   059 01   104 05
015 01   060 05   105 65
016 04   061 65   106 43
017 81   062 43   107 01
018 42   063 01   108 07
019 01   064 09   109 85
020 05   065 95   110 43
021 81   066 42   111 01
022 42   067 00   112 06
023 01   068 07   113 65
024 06   069 81   114 43
025 81   070 46   115 01
026 46   071 13   116 08
027 11   072 43   117 85
028 42   073 01   118 43
029 01   074 03   119 01
030 07   075 65   120 04
031 81   076 43   121 65
032 42   077 01   122 43
033 01   078 07   123 01
034 08   079 85   124 09
035 81   080 43   125 95
036 42   081 01   126 42
037 01   082 02   127 00
038 09   083 65   128 09
039 81   084 43   129 81
040 46   085 01
041 12   086 08
042 43   087 85
043 01   088 43
044 01   089 01
```

SAMPLE PROBLEM: 023 STRAIN-STRESS RELATIONS NO: 023

1. Given

$$e_i = \begin{Bmatrix} 2.1171 \\ 5.2064 \\ 2.0921 \end{Bmatrix} \text{ mm/m}$$

$$Q_{ij} = \begin{bmatrix} 181.811 & 2.897 & 0 \\ 2.897 & 10.346 & 0 \\ 0 & 0 & 7.170 \end{bmatrix} \text{ (GPa)}$$

(For T-300/5208)

$$\sigma_i = \begin{Bmatrix} 400 \\ 60 \\ 15 \end{Bmatrix} \text{ (MPa)}$$

2. Given

$$e_i = \begin{Bmatrix} .0243 \\ 1.9263 \\ 4.1841 \end{Bmatrix} \text{ mm/m}$$

$A_{ij}$ (GPa) (for T-300/5208)

$$\sigma_i = \begin{Bmatrix} 10 \\ 20 \\ 30 \end{Bmatrix} \text{ MPa}$$

TITLE: MODULUS TO COMPLIANCE NO: 027

| | | | | |
|---|---|---|---|---|
| A' $Q_{ij}$ | B' | C' | D' | E' |
| A $\lvert Q \rvert$ | B | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $S_{11}$ | 02 $S_{22}$ | 03 $S_{12}$ | 04 $S_{66}$ | 05 $S_{16}$ | 06 $S_{26}$ |
| 07 | 08 | 09 | 10 $\lvert Q \rvert$ | | |
| 11 $Q_{11}$ | 12 $Q_{22}$ | 13 $Q_{12}$ | 14 $Q_{66}$ | 15 $Q_{16}$ | 16 $Q_{26}$ |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input $Q_{ij}$ | 1 | 027 A&B | $Q_{11}$ | A' | $Q_{11}$ |
| | | | $Q_{22}$ | RUN | $Q_{22}$ |
| | | | $Q_{12}$ | RUN | $Q_{12}$ |
| | | | $Q_{66}$ | RUN | $Q_{66}$ |
| | | | $Q_{16}$ | RUN | $Q_{16}$ |
| | | | $Q_{26}$ | RUN | $Q_{26}$ |
| Compute $S_{ij}$ | 2 | | | A | $S_{26}$ |
| | | | | RCL 01 | $S_{11}$ |
| | | | | RCL 02 | $S_{22}$ |
| | | | | RCL 03 | $S_{12}$ |
| | | | | RCL 04 | $S_{66}$ |
| | | | | RCL 05 | $S_{16}$ |
| | | | | RCL 06 | $S_{26}$ |

**PROGRAM TITLE:** MODULUS TO COMPLIANCE **NO:** 027

**FORMULAS:**

**In-Plane Modulus and Compliance**

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} \quad \text{or} \quad N_i = A_{ij}\, e_j^o \qquad (203)$$

$$\text{Let Compliance} = {}_{ij} = [A_{ij}]^{-1} \quad \text{or} \quad {}_{ij}A_{jk} = \delta_{ik} \qquad (204)$$

$$\begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or} \quad e_i^o = a_{ij}N_j \qquad (205)$$

where

$$a_{11} = \frac{1}{\Delta}(A_{22}A_{66}-A_{26}^2), \qquad a_{22} = \frac{1}{\Delta}(A_{11}A_{66}-A_{16}^2)$$

$$a_{12} = \frac{1}{\Delta}(A_{16}A_{26}-A_{12}A_{66}), \qquad a_{66} = \frac{1}{\Delta}(A_{11}A_{22}-A_{12}^2) \qquad (206)$$

$$a_{16} = \frac{1}{\Delta}(A_{12}A_{26}-A_{22}A_{16}), \qquad a_{26} = \frac{1}{\Delta}(A_{12}A_{16}-A_{11}A_{26})$$

$$\Delta = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}$$

**In-Plane Engineering Constants:**

$$E_{11}^o = \frac{1}{ha_{11}}, \quad E_{22}^o = \frac{1}{ha_{22}}, \quad \nu_{12}^o = -\frac{a_{12}}{a_{11}}, \quad \nu_{21}^o = -\frac{a_{12}}{a_{22}}, \quad G_{12}^o = \frac{1}{ha_{66}} \qquad (207)$$

**PROGRAM TITLE:** MODULUS TO COMPLIANCE **NO:** 027

**PROGRAM LISTING:**

```
000 46   045 01   090 04   135 01   180 01
001 16   046 06   091 75   136 03   181 05
002 42   047 65   092 43   137 40   182 65
003 01   048 43   093 01   138 95   183 43
004 01   049 01   094 06   139 55   184 01
005 81   050 05   095 40   140 43   185 02
006 42   051 65   096 95   141 01   186 95
007 01   052 02   097 55   142 00   187 55
008 02   053 75   098 43   143 95   188 43
009 81   054 43   099 01   144 42   189 01
010 42   055 01   100 00   145 00   190 00
011 01   056 05   101 95   146 04   191 95
012 03   057 40   102 42   147 43   192 42
013 81   058 65   103 00   148 01   193 00
014 42   059 43   104 01   149 05   194 05
015 01   060 01   105 43   150 65   195 43
016 04   061 02   106 01   151 43   196 01
017 81   062 75   107 01   152 01   197 03
018 42   063 43   108 65   153 06   198 65
019 01   064 01   109 43   154 75   199 43
020 05   065 03   110 01   155 43   200 01
021 81   066 40   111 04   156 01   201 05
022 42   067 65   112 75   157 03   202 75
023 01   068 43   113 43   158 65   203 43
024 06   069 01   114 01   159 43   204 01
025 81   070 04   115 05   160 01   205 01
026 46   071 75   116 40   161 04   206 65
027 11   072 43   117 95   162 95   207 43
028 43   073 01   118 55   163 55   208 01
029 01   074 06   119 43   164 43   209 06
030 01   075 40   120 01   165 01   210 95
031 65   076 65   121 00   166 00   211 55
032 43   077 43   122 95   167 95   212 43
033 01   078 01   123 42   168 42   213 01
034 02   079 01   124 00   169 00   214 00
035 65   080 95   125 02   170 03   215 95
036 43   081 42   126 43   171 43   216 42
037 01   082 01   127 01   172 01   217 00
038 04   083 00   128 01   173 03   218 06
039 85   084 43   129 65   174 65   219 81
040 43   085 01   130 43   175 43
041 01   086 02   131 01   176 01
042 03   087 65   132 02   177 06
043 65   088 43   133 75   178 75
044 43   089 01   134 43   179 43
```

SAMPLE PROBLEM: 027 MODULUS TO COMPLIANCE NO: 027

$$Q_{ij} = \begin{bmatrix} 181.8111 & 2.8969 & 0 \\ 2.8969 & 10.3462 & 0 \\ 0 & 0 & 7.1700 \end{bmatrix} \text{(MPa)}$$

$$S_{ij} = \begin{bmatrix} 5.5249 & -1.5469 & 0 \\ -1.5469 & 97.0870 & 0 \\ 0 & 0 & 139.4700 \end{bmatrix} (\text{TPa})^{-1}$$

TITLE: COMPLIANCE TO MODULUS NO: 031

| | | | | |
|---|---|---|---|---|
| A' $S_{ij}$ | B' | C' | D' | E' |
| A $\|S\|$ | B | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $S_{11}$ | 02 $S_{22}$ | 03 $S_{12}$ | 04 $S_{66}$ | 05 $S_{16}$ | 06 $S_{26}$ |
| 07 | 08 | 09 | 10 $\|S\|$ | | |
| 11 $Q_{11}$ | 12 $Q_{22}$ | 13 $Q_{12}$ | 14 $Q_{66}$ | 15 $Q_{16}$ | 16 $Q_{26}$ |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input $S_{ij}$ | 1 | 031 A& B | $S_{11}$ | A' | $S_{11}$ |
| | | | $S_{22}$ | RUN | $S_{12}$ |
| | | | $S_{12}$ | RUN | $S_{12}$ |
| | | | $S_{66}$ | RUN | $S_{66}$ |
| | | | $S_{16}$ | RUN | $S_{16}$ |
| | | | $S_{26}$ | RUN | $S_{26}$ |
| Compute $Q_{ij}$ | 2 | | | A | $Q_{11}$ |
| | | | | RCL 11 | $Q_{11}$ |
| | | | | RCL 12 | $Q_{22}$ |
| | | | | RCL 13 | $Q_{12}$ |
| | | | | RCL 14 | $Q_{66}$ |
| | | | | RCL 15 | $Q_{16}$ |
| | | | | RCL 16 | $Q_{26}$ |

**PROGRAM TITLE:** COMPLIANCE TO MODULUS **NO:** 031

**PROGRAM LISTING:**

```
000 46   045 00   090 02   135 00   180 03
001 16   046 06   091 75   136 03   181 75
002 42   047 65   092 43   137 65   182 43
003 00   048 43   093 00   138 43   183 00
004 01   049 00   094 03   139 00   184 01
005 81   050 05   095 40   140 04   185 65
006 42   051 65   096 95   141 95   186 43
007 00   052 02   097 55   142 55   187 00
008 02   053 75   098 43   143 43   188 06
009 81   054 43   099 01   144 01   189 95
010 42   055 00   100 00   145 00   190 55
011 00   056 05   101 95   146 95   191 43
012 03   057 40   102 42   147 42   192 01
013 81   058 65   103 01   148 01   193 00
014 42   059 43   104 04   149 03   194 95
015 00   060 00   105 43   150 43   195 42
016 04   061 02   106 00   151 00   196 01
017 81   062 75   107 01   152 03   197 06
018 42   063 43   108 65   153 65   198 43
019 00   064 00   109 43   154 43   199 00
020 05   065 03   110 00   155 00   200 02
021 81   066 40   111 04   156 06   201 65
022 42   067 65   112 75   157 75   202 43
023 00   068 43   113 43   158 43   203 00
024 06   069 00   114 00   159 00   204 04
025 81   070 04   115 05   160 05   205 75
026 46   071 75   116 40   161 65   206 43
027 11   072 43   117 95   162 43   207 00
028 43   073 00   118 55   163 00   208 06
029 00   074 06   119 43   164 02   209 40
030 01   075 40   120 01   165 95   210 95
031 65   076 65   121 00   166 55   211 55
032 43   077 43   122 95   167 43   212 43
033 00   078 00   123 42   168 01   213 01
034 02   079 01   124 01   169 00   214 00
035 65   080 95   125 02   170 95   215 95
036 43   081 42   126 43   171 42   216 42
037 00   082 01   127 00   172 01   217 01
038 04   083 00   128 05   173 05   218 01
039 85   084 43   129 65   174 43   219 81
040 43   085 00   130 43   175 00
041 00   086 01   131 00   176 05
042 03   087 65   132 06   177 65
043 65   088 43   133 75   178 43
044 43   089 00   134 43   179 00
```

TITLE: INVARIANTS OF ELASTICITY NO: 033

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A Initialize | B | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $\frac{1}{m}$ | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $Q_{11}$ | 06 $Q_{22}$ |
| 07 | 08 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 $Q_{12}$ | 16 $S_{12}$ |
| 17 | 18 | 19 | | | |
| 98 $E_L$ | 99 $E_T$ | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input Engineering Constants | 1 | | $E_L$ | STO 98 | $E_L$ |
| | | | $E_T$ | STO 99 | $E_T$ |
| | | | $\nu_{LT}$ | STO 09 | $\nu_{LT}$ |
| | | | $G_{LT}$ | STO 10 | $G_{LT}$ |
| Compute invariants | 2 | 033 A&B | | A | $R_{2S}$ |
| | | | | RCL 11 | $I_{1S}$ |
| | | | | RCL 12 | $I_{2S}$ |
| | | | | RCL 13 | $R_{1S}$ |
| | | | | RCL 14 | $R_{2S}$ |
| | | | | RCL 01 | $I_{1Q}$ |
| | | | | RCL 02 | $I_{2Q}$ |
| | | | | RCL 03 | $R_{1Q}$ |
| | | | | RCL 04 | $R_{2Q}$ |

PROGRAM TITLE: INVARIANTS OF ELASTICITY NO: 033

FORMULAS:

TABLE 10 COMPLIANCE AND MODULUS IN TERMS OF ELASTIC SYMMETRIES

| Symmetry (No Indep. Const) | Compliance Matrix $S_{ij}$ $(TPa)^{-1}$ | Modulus Matrix $Q_{ij}$ (MPa) |
|---|---|---|
| Anisotropic (6) or Generally Orthotropic (4) | $\begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & S_{16} \\ -\frac{\nu_{21}}{E_{22}} & \frac{1}{E_{22}} & S_{26} \\ S_{61} & S_{62} & \frac{1}{G_{12}} \end{bmatrix}$ | $\begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}$ |
| Specially Orthotropic (4) | $\begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{LT}}{E_L} & 0 \\ -\frac{\nu_{TL}}{E_T} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix}$ | $\begin{bmatrix} mE_L & \nu_{TL} mE_L & 0 \\ \nu_{LT} mE_T & mE_T & 0 \\ 0 & 0 & G_{LT} \end{bmatrix}$ |

$$m = \frac{1}{1-\nu_{LT}\nu_{TL}} = \frac{1}{1-\nu_{LT}^2 \frac{E_T}{E_L}}$$

$$I_1 = \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12}) \qquad (89)$$

$$I_2 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \qquad (90)$$

$$R_1 = \frac{1}{2}\sqrt{(-Q_{11} + Q_{22})^2} \qquad (91)$$

$$R_2 = \frac{1}{8}\sqrt{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})^2} \qquad (92)$$

$$I_1 = \frac{1}{4}(S_{11} + S_{22} + 2S_{12}) \qquad (81)$$

$$I_2 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} + S_{66}) \qquad (82)$$

$$R_1 = \frac{1}{2}\sqrt{(-S_{11} + S_{22})^2} \qquad (83)$$

$$R_2 = \frac{1}{8}\sqrt{(S_{11} + S_{22} - 2S_{12} - S_{66})^2} \qquad (84)$$

**PROGRAM TITLE:** INVARIANTS OF ELASTICITY **NO:** 033

**PROGRAM LISTING:**

```
000 46   045 42   090 02   135 54   180 09
001 11   046 01   091 43   136 42   181 09
002 01   047 05   092 00   137 01   182 20
003 75   048 65   093 05   138 06   183 75
004 43   049 02   094 75   139 65   184 43
005 00   050 85   095 43   140 02   185 09
006 09   051 43   096 00   141 95   186 08
007 40   052 00   097 06   142 55   187 20
008 65   053 05   098 95   143 04   188 95
009 43   054 85   099 55   144 95   189 55
010 09   055 43   100 02   145 51   190 02
011 09   056 00   101 95   146 87   191 95
012 55   057 06   102 42   147 42   192 51
013 43   058 95   103 00   148 01   193 87
014 09   059 55   104 03   149 01   194 42
015 08   060 04   105 43   150 43   195 01
016 95   061 95   106 00   151 09   196 03
017 42   062 42   107 02   152 08   197 43
018 00   063 00   108 75   153 20   198 01
019 00   064 01   109 43   154 85   199 00
020 55   065 43   110 01   155 43   200 20
021 43   066 00   111 00   156 09   201 51
022 09   067 05   112 95   157 09   202 87
023 08   068 85   113 42   158 20   203 55
024 95   069 43   114 00   159 75   204 04
025 20   070 00   115 04   160 02   205 75
026 42   071 06   116 43   161 65   206 43
027 00   072 75   117 09   162 43   207 01
028 05   073 02   118 08   163 01   208 02
029 43   074 65   119 20   164 06   209 95
030 09   075 43   120 85   165 85   210 42
031 09   076 01   121 43   166 43   211 01
032 55   077 05   122 09   167 01   212 04
033 43   078 85   123 09   168 00   213 81
034 00   079 04   124 20   169 20   214 46
035 00   080 65   125 85   170 95   215 87
036 95   081 43   126 53   171 55   216 65
037 42   082 01   127 43   172 08   217 01
038 00   083 00   128 00   173 95   218 00
039 06   084 95   129 09   174 51   219 00
040 65   085 55   130 55   175 87   220 00
041 43   086 08   131 43   176 42   221 95
042 00   087 95   132 09   177 01   222 56
043 09   088 42   133 08   178 02
044 95   089 00   134 94   179 43
```

SAMPLE PROBLEM: 033 INVARIANTS OF ELASTICITY NO: 033

Given:

| Material | Type | $E_L$ (GPa) | $E_T$ (GPa) | $\nu_{LT}$ | $G_{LT}$ (GPa) | Fiber Vol. $v_f$ | Specific Gravity |
|---|---|---|---|---|---|---|---|
| B/Ep | B(4) 5505 | 204 | 18.5 | .23 | 5.79 | .5 | 2.0 |
| Gr/Ep | T-300 5208 | 181 | 10.3 | .28 | 7.17 | .70 | 1.60 |
| Gl/Ep | Scotchply 1002 | 38.6 | 8.27 | .26 | 4.14 | .45 | 1.8 |

### INVARIANTS FOR COMPLIANCE MATRIX

| Material | $I_{1S}$ | $I_{2S}$ | $R_{1S}$ | $R_{2S}$ | $\delta_1 = \delta_2$ |
|---|---|---|---|---|---|
| B/Ep | 14.18 | 29.24 | 24.58 | 13.94 | 0 |
| Gr/Ep | 24.88 | 30.65 | 45.78 | 4.22 | 0 |
| Gl/Ep | 33.3 | 50.2 | 47.5 | 10.2 | 0 |

### INVARIANTS FOR MODULUS MATRIX

| Material | $I_{1Q}$ | $I_{2Q}$ | $R_{1Q}$ | $R_{2Q}$ | $\delta_1 = \delta_2$ |
|---|---|---|---|---|---|
| B/Ep | 58.03 | 29.77 | 93.20 | 23.98 | 0 |
| Gr/Ep | 49.49 | 26.88 | 85.73 | 19.71 | 0 |
| Gl/Ep | 13.0 | 7.47 | 15.4 | 3.33 | 0 |

TITLE: COMPLIANCE COMPONENTS NO: 037

| | | | | |
|---|---|---|---|---|
| A' | B' $S_{16}$ | C' $S_{26}$ | D' | E' |
| A Invariants | B $S_{11}$ | C $S_{22}$ | D $S_{12}$ | E $S_{66}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $\theta$ | | | | | |
| 01 $S_{11}$ | 02 $S_{22}$ | 03 $S_{12}$ | 04 $S_{66}$ | 05 $S_{16}$ | 06 $S_{26}$ |
| 07 | 08 | 09 | 10 | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 | 16 |
| 17 $\delta_{1S}$ | 18 $\delta_{2S}$ | 19 $2(\theta-\delta_1)$ | | | |
| 98 $4(\theta-\delta_2)$ | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input Invariants | 1 | 037 A&B | $I_{1S}$ | A | $I_{1S}$ |
| | | | $I_{2S}$ | RUN | $I_{2S}$ |
| | | | $R_{1S}$ | RUN | $R_{1S}$ |
| | | | $R_{2S}$ | RUN | $R_{2S}$ |
| | | | $\delta_{1S}$ | RUN | $\delta_{1S}$ |
| | | | $\delta_{2S}$ | RUN | $\delta_{2S}$ |
| Input $\theta$ & Compute | 2 | | $\theta$ (deg) | B | $S_{11}$ |
| | | | | C | $S_{22}$ |
| | | | | D | $S_{12}$ |
| | | | | E | $S_{66}$ |
| | | | | B' | $S_{16}$ |
| | | | | C' | $S_{26}$ |

PROGRAM TITLE: COMPLIANCE COMPONENTS NO: 037

FORMULAS:

TABLE 17 TRANSFORMATION OF COMPLIANCE

| | $I_1$ | $I_2$ | $R_1$ | $R_2$ |
|---|---|---|---|---|
| $S'_{11}$ | 1 | 1 | $-\cos 2(\theta - \delta_1)$ | $-\cos 4(\theta - \delta_2)$ |
| $S'_{22}$ | 1 | 1 | $\cos 2(\theta - \delta_1)$ | $-\cos 4(\theta - \delta_2)$ |
| $S'_{12}$ | 1 | -1 | 0 | $\cos 4(\theta - \delta_2)$ |
| $S'_{66}$ | 0 | 4 | 0 | $4 \cos 4(\theta - \delta_2)$ |
| $S'_{16}$ | 0 | 0 | $\sin 2(\theta - \delta_1)$ | $2 \sin 4(\theta - \delta_2)$ |
| $S'_{26}$ | 0 | 0 | $\sin 2(\theta - \delta_1)$ | $-2 \sin 4(\theta - \delta_2)$ |

$$I_1 = \frac{1}{4}(S_{11} + S_{22} + 2S_{12}) \tag{82}$$

$$I_2 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} + S_{66}) \tag{83}$$

$$R_1 = \frac{1}{2}\sqrt{(-S_{11} + S_{22})^2 + (S_{16} + S_{26})^2} \tag{84}$$

$$R_2 = \frac{1}{8}\sqrt{(S_{11} + S_{22} - 2S_{12} - S_{66})^2 + 4(S_{26} - S_{16})^2} \tag{85}$$

$$\tan 2\delta_1 = \frac{S_{16} + S_{26}}{S_{11} - S_{22}} \tag{86}$$

$$\tan 4\delta_2 = \frac{2(S_{16} - S_{26})}{S_{11} + S_{22} - 2S_{12} - S_{66}} \tag{87}$$

For orthotropic material

$$\delta_1 = \delta_2 \tag{88}$$

For anisotropic material

$$\delta_1 \neq \delta_2 \tag{89}$$

**PROGRAM TITLE:** COMPLIANCE COMPONENTS **NO:** 037

**PROGRAM LISTING:**

```
000 46   045 01   090 06   135 85   180 14
001 11   046 03   091 43   136 43   181 43
002 42   047 85   092 09   137 01   182 00
003 01   048 53   093 08   138 09   183 03
004 01   049 53   094 33   139 33   184 81
005 81   050 43   095 65   140 65   185 46
006 42   051 00   096 43   141 43   186 15
007 01   052 00   097 01   142 01   187 43
008 02   053 75   098 04   143 03   188 00
009 81   054 43   099 85   144 75   189 04
010 42   055 01   100 43   145 43   190 81
011 01   056 08   101 01   146 09   191 46
012 03   057 54   102 01   147 08   192 17
013 81   058 65   103 75   148 33   193 43
014 42   059 04   104 43   149 65   194 00
015 01   060 54   105 01   150 43   195 05
016 04   061 42   106 02   151 01   196 81
017 81   062 09   107 95   152 04   197 46
018 42   063 08   108 42   153 95   198 18
019 01   064 32   109 00   154 42   199 43
020 07   065 65   110 03   155 00   200 00
021 81   066 02   111 75   156 02   201 06
022 42   067 65   112 43   157 75   202 81
023 01   068 43   113 01   158 43
024 08   069 01   114 01   159 01
025 81   070 04   115 85   160 09
026 46   071 95   116 02   161 33
027 12   072 42   117 65   162 65
028 42   073 00   118 43   163 43
029 00   074 05   119 01   164 01
030 00   075 94   120 02   165 03
031 75   076 85   121 95   166 65
032 43   077 43   122 65   167 02
033 01   078 01   123 04   168 95
034 07   079 09   124 95   169 42
035 95   080 32   125 42   170 00
036 65   081 65   126 00   171 01
037 02   082 43   127 04   172 81
038 95   083 01   128 43   173 46
039 42   084 03   129 01   174 13
040 01   085 65   130 01   175 43
041 09   086 02   131 85   176 00
042 32   087 95   132 43   177 02
043 65   088 42   133 01   178 81
044 43   089 00   134 02   179 46
```

## SAMPLE PROBLEM: 037 COMPLIANCE COMPONENTS NO: 037

Given:

| Material | Type | $I_{1S}$ | $I_{2S}$ | $R_{1S}$ | $R_{2S}$ | $\delta_1=\delta_2$ |
|---|---|---|---|---|---|---|
| B/Ep | B(4) 5505 | 14.18 | 29.24 | 24.58 | 13.94 | 0 |
| Gr/Ep | T-300 5208 | 24.88 | 30.65 | 45.78 | 4.22 | 0 |
| Gl/Ep | Scotchply 1002 | 33.3 | 50.2 | 47.5 | 10.2 | 0 |

| | T-300 / 5208 | | | |
|---|---|---|---|---|
| | $S_{ij}$ (TPa)$^{-1}$ | | | Engr. Const. (GPa) |
| 0° | 5.52 | -1.55 | 0 | $E'_{11}$ = 181 |
| | | | | $E'_{22}$ = 10.3 |
| | -1.55 | 97.09 | 0 | $\nu'_{12}$ = .28 |
| | | | | $\nu'_{21}$ = .016 |
| | 0 | 0 | 139.47 | $G'_{12}$ = 7.2 |
| 30° | 34.75 | -7.88 | 46.96 | $E'_{11}$ = 28.78 |
| | | | | $E'_{22}$ = 12.42 |
| | -7.88 | 80.53 | 32.34 | $\nu'_{12}$ = .23 |
| | | | | $\nu'_{21}$ = .10 |
| | 46.96 | 32.34 | 114.15 | $G'_{12}$ = 8.76 |
| 45° | 59.75 | -9.99 | 45.78 | $E'_{11}$ = 16.74 |
| | | | | $E'_{22}$ = 16.74 |
| | -9.99 | 59.75 | 45.78 | $\nu'_{12}$ = .17 |
| | | | | $\nu'_{21}$ = .17 |
| | 45.78 | 45.78 | 105.71 | $G'_{12}$ = 9.46 |

TITLE: MODULUS COMPONENTS NO: 041

| | | | | |
|---|---|---|---|---|
| A' | B' $Q_{16}$ | C' $Q_{26}$ | D' | E' |
| A Invariants | B $Q_{11}$ | C $Q_{22}$ | D $Q_{12}$ | E $Q_{66}$ |

| | | | | | |
|---|---|---|---|---|---|
| 00 $\theta$ | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 | 06 |
| 07 $\delta_{1Q}$ | 08 $\delta_{2Q}$ | 09 $2(\theta-\delta_1)$ | 10 | | |
| 11 $Q_{11}$ | 12 $Q_{22}$ | 13 $Q_{12}$ | 14 $Q_{66}$ | 15 $Q_{16}$ | 16 $Q_{26}$ |
| 17 | 18 | 19 | | | |
| 98 | 99 $4(\theta-\delta_2)$ | | | | |

REGISTER

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input Invariants | 1 | 041 A&B | $I_{1Q}$ | A | $I_{1Q}$ |
| | | | $I_{2Q}$ | RUN | $I_{2Q}$ |
| | | | $R_{1Q}$ | RUN | $R_{1Q}$ |
| | | | $R_{2Q}$ | RUN | $R_{2Q}$ |
| | | | $\delta_{1Q}$ | RUN | $\delta_{1Q}$ |
| | | | $\delta_{2Q}$ | RUN | $\delta_{2Q}$ |
| Input $\theta$ & Compute | 2 | | $\theta$(deg) | B | $Q_{11}$ |
| | | | | C | $Q_{22}$ |
| | | | | D | $Q_{12}$ |
| | | | | E | $Q_{66}$ |
| | | | | B' | $Q_{16}$ |
| | | | | C' | $Q_{26}$ |

**PROGRAM TITLE:** MODULUS COMPONENTS **NO:** 041

**FORMULAS:**

TABLE 20 TRANSFORMATION OF MODULUS

| | $I_1$ | $I_2$ | $R_1$ | $R_2$ |
|---|---|---|---|---|
| $Q'_{11}$ | 1 | 1 | $\cos 2(\theta - \delta_1)$ | $\cos 4(\theta - \delta_2)$ |
| $Q'_{22}$ | 1 | 1 | $-\cos 2(\theta - \delta_1)$ | $\cos 4(\theta - \delta_2)$ |
| $Q'_{12}$ | 1 | -1 | 0 | $-\cos 4(\theta - \delta_2)$ |
| $Q'_{66}$ | 0 | 1 | 0 | $-\cos 4(\theta - \delta_2)$ |
| $Q'_{16}$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta - \delta_1)$ | $-\sin 4(\theta - \delta_2)$ |
| $Q_{26}$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta - \delta_1)$ | $\sin 4(\theta - \delta_2)$ |

$$I_1 = \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12}) \tag{90}$$

$$I_2 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \tag{91}$$

$$R_1 = \frac{1}{2}\sqrt{(-Q_{11} + Q_{22})^2 + 4(Q_{16} + Q_{26})^2} \tag{92}$$

$$R_2 = \frac{1}{8}\sqrt{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})^2 + 16(Q_{16} - Q_{26})^2} \tag{93}$$

$$\tan 2\delta_1 = + \frac{2(Q_{16} + Q_{26})}{Q_{11} - Q_{22}} \tag{94}$$

$$\tan 4\delta_2 = + \frac{4(Q_{16} - Q_{26})}{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}} \tag{95}$$

$\delta_1 = \delta_2$ for orthotropic materials

$\delta_1 \neq \delta_2$ for anisotropic materials

**PROGRAM TITLE:** MODULUS COMPONENTS **NO:** 041

**PROGRAM LISTING:**

```
000 46   045 94   090 06   135 65   180 15
001 11   046 65   091 43   136 43   181 43
002 42   047 43   092 09   137 00   182 01
003 00   048 00   093 09   138 03   183 04
004 01   049 03   094 33   139 85   184 81
005 81   050 75   095 94   140 43   185 46
006 42   051 53   096 65   141 00   186 17
007 00   052 53   097 43   142 01   187 43
008 02   053 43   098 00   143 85   188 01
009 81   054 00   099 04   144 43   189 05
010 42   055 00   100 85   145 00   190 81
011 00   056 75   101 43   146 02   191 46
012 03   057 43   102 00   147 95   192 18
013 81   058 00   103 02   148 42   193 43
014 42   059 08   104 95   149 01   194 01
015 00   060 54   105 42   150 02   195 06
016 04   061 65   106 01   151 85   196 81
017 81   062 04   107 04   152 43
018 42   063 54   108 85   153 00
019 00   064 42   109 43   154 09
020 07   065 09   110 00   155 33
021 81   066 09   111 01   156 65
022 42   067 32   112 75   157 43
023 00   068 65   113 02   158 00
024 08   069 43   114 65   159 03
025 81   070 00   115 43   160 65
026 46   071 04   116 00   161 02
027 12   072 95   117 02   162 95
028 42   073 42   118 95   163 42
029 00   074 01   119 42   164 01
030 00   075 05   120 01   165 01
031 75   076 85   121 03   166 81
032 43   077 43   122 43   167 46
033 00   078 09   123 09   168 13
034 07   079 09   124 09   169 43
035 95   080 32   125 33   170 01
036 65   081 65   126 65   171 02
037 02   082 43   127 43   172 81
038 95   083 00   128 00   173 46
039 42   084 04   129 04   174 14
040 00   085 65   130 75   175 43
041 09   086 02   131 43   176 01
042 32   087 95   132 00   177 03
043 55   088 42   133 09   178 81
044 02   089 01   134 33   179 46
```

## SAMPLE PROBLEM: 041 MODULUS COMPONENTS NO: 041

Given:

| Material | Type | $I_{1Q}$ | $I_{2Q}$ | $R_{1Q}$ | $R_{2Q}$ | $\delta_1 = \delta_2$ |
|---|---|---|---|---|---|---|
| B/Ep | B(4) / 5505 | 58.03 | 29.77 | 93.20 | 23.98 | 0 |
| Gr/Ep | T-300 5208 | 49.49 | 26.88 | 85.73 | 19.71 | 0 |
| Gl/Ep | Scotchply 1002 | 13.0 | 7.47 | 15.4 | 3.33 | 0 |

|  | T-300 / 5208 |  |  | B(4) / 5504 |  |  |
|---|---|---|---|---|---|---|
|  | $Q'_{ij}$ |  |  | $Q'_{ij}$ |  |  |
| $0^\circ$ | 181.81 | 42.90 | 0 | 204.98 | 4.28 | 0 |
|  | 42.90 | 10.35 | 0 | 4.28 | 18.59 | 0 |
|  | 0 | 0 | 7.17 | 0 | 0 | 5.79 |
| $30^\circ$ | 109.38 | 32.46 | -54.19 | 122.41 | 40.25 | -61.13 |
|  | 32.46 | 23.65 | -20.05 | 40.25 | 29.21 | -19.59 |
|  | -54.19 | -20.05 | 36.74 | -61.13 | -19.59 | 41.76 |
| $45^\circ$ | 56.66 | 42.32 | -42.89 |  |  |  |
|  | 42.32 | 56.66 | -42.87 |  |  |  |
|  | -42.87 | -42.87 | 46.59 |  |  |  |

TITLE: PLY DATA: T-300/5208 Gr/Ep NO: 045

| | | | | |
|---|---|---|---|---|
| A' X, $e_L$ | B' X', $e_L'$ | C' Y, $e_T$ | D' Y', $e_T'$ | E' S, $e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 S | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 X | 16 X' |
| 17 0 | 18 0 | 19 0 | | | |
| 98 Y | 99 Y' | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 045 A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | X |
| | | | | RUN | $e_L$ |
| | | | | B' | X' |
| | | | | RUN | $e_L'$ |
| | | | | C' | Y |
| | | | | RUN | $e_T$ |
| | | | | D' | Y' |
| | | | | RUN | $e_T'$ |
| | | | | E' | S |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA: T300/5208 Gr/Ep NO: 045

PROGRAM LISTING:

```
000 46   045 02   090 03   135 08   180 43
001 11   046 08   091 04   136 42   181 00
002 01   047 05   092 93   137 00   182 05
003 08   048 93   093 02   138 07   183 95
004 01   049 07   094 02   139 43   184 81
005 42   050 03   095 00   140 00   185 46
006 00   051 02   096 42   141 05   186 18
007 05   052 42   097 01   142 81   187 43
008 01   053 00   098 04   143 46   188 09
009 00   054 03   099 01   144 12   189 08
010 93   055 01   100 05   145 43   190 81
011 03   056 09   101 00   146 00   191 55
012 42   057 93   102 00   147 06   192 43
013 00   058 07   103 42   148 81   193 00
014 06   059 01   104 01   149 46   194 06
015 93   060 00   105 05   150 13   195 95
016 02   061 42   106 01   151 43   196 81
017 08   062 00   107 05   152 00   197 46
018 42   063 04   108 00   153 09   198 19
019 00   064 02   109 00   154 81   199 43
020 09   065 04   110 42   155 46   200 09
021 07   066 93   111 01   156 14   201 09
022 93   067 09   112 06   157 43   202 81
023 01   068 08   113 04   158 01   203 55
024 07   069 00   114 00   159 00   204 43
025 42   070 42   115 42   160 81   205 00
026 01   071 01   116 09   161 46   206 06
027 00   072 01   117 08   162 16   207 95
028 04   073 03   118 02   163 43   208 81
029 09   074 00   119 04   164 01   209 46
030 93   075 93   120 06   165 05   210 10
031 04   076 06   121 42   166 81   211 43
032 08   077 04   122 09   167 55   212 00
033 08   078 07   123 09   168 43   213 00
034 42   079 42   124 06   169 00   214 81
035 00   080 01   125 08   170 05   215 55
036 01   081 02   126 42   171 95   216 43
037 02   082 04   127 00   172 81   217 01
038 06   083 05   128 00   173 46   218 00
039 93   084 93   129 00   174 17   219 95
040 08   085 07   130 42   175 43   220 81
041 08   086 08   131 00   176 01
042 00   087 01   132 07   177 06
043 42   088 42   133 42   178 81
044 00   089 01   134 00   179 55
```

TITLE: PLY DATA: B/5505 B/Ep NO: 047

| | | | | |
|---|---|---|---|---|
| A' $X$, $e_L$ | B' $X'$, $e_L'$ | C' $Y$, $e_T$ | D' $Y'$, $e_T'$ | E' $S$, $e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $S$ | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 $X$ | 16 $X'$ |
| 17 0 | 18 0 | 19 0 | | | |
| 98 $Y$ | 99 $Y'$ | | | | |

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 047 A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | $X$ |
| | | | | RUN | $e_L$ |
| | | | | B' | $X'$ |
| | | | | RUN | $e_L'$ |
| | | | | C' | $Y$ |
| | | | | RUN | $e_T$ |
| | | | | D' | $Y'$ |
| | | | | RUN | $e_T$ |
| | | | | E' | $S$ |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA: B/5505 B/Ep NO: 047

PROGRAM LISTING:

```
000 46   045 00   090 00   135 81   180 43
001 11   046 03   091 42   136 46   181 09
002 02   047 02   092 06   137 14   182 09
003 00   048 04   093 09   138 43   183 81
004 04   049 42   094 06   139 01   184 55
005 42   050 00   095 01   140 00   185 43
006 00   051 04   096 42   141 81   186 00
007 05   052 01   097 09   142 46   187 06
008 01   053 04   098 08   143 16   188 95
009 08   054 93   099 02   144 43   189 81
010 93   055 02   100 00   145 01   190 46
011 05   056 42   101 02   146 05   191 10
012 42   057 01   102 42   147 81   192 43
013 00   058 01   103 09   148 55   193 00
014 06   059 02   104 09   149 43   194 00
015 93   060 09   105 06   150 00   195 81
016 02   061 93   106 07   151 05   196 55
017 03   062 02   107 42   152 95   197 43
018 42   063 42   108 00   153 81   198 01
019 01   064 01   109 00   154 46   199 00
020 06   065 02   110 00   155 17   200 95
021 05   066 02   111 42   156 43   201 81
022 93   067 04   112 00   157 06
023 07   068 93   113 07   158 09
024 09   069 06   114 42   159 81
025 42   070 42   115 00   160 55
026 01   071 01   116 08   161 43
027 00   072 03   117 42   162 00
028 05   073 01   118 00   163 05
029 08   074 03   119 09   164 95
030 42   075 93   120 43   165 81
031 00   076 09   121 00   166 46
032 01   077 42   122 05   167 18
033 02   078 01   123 81   168 43
034 09   079 04   124 46   169 09
035 93   080 01   125 12   170 08
036 08   081 02   126 43   171 81
037 42   082 06   127 00   172 55
038 00   083 00   128 06   173 43
039 02   084 42   129 81   174 00
040 09   085 01   130 46   175 06
041 03   086 05   131 13   176 95
042 93   087 02   132 43   177 81
043 02   088 05   133 01   178 46
044 42   089 00   134 06   179 19
```

TITLE: PLY DATA: SCOTCHPLY 1002 NO: 049

| | | | | |
|---|---|---|---|---|
| A' $X, e_L$ | B' $X', e_L'$ | C' $Y, e_T$ | D' $Y', e_T'$ | E' $S, e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $S$ | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 $X$ | 16 $X'$ |
| 17 0 | 18 0 | 19 0 | | | |
| 98 $Y$ | 99 $Y'$ | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 049 A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | $X$ |
| | | | | RUN | $e_L$ |
| | | | | B' | $X'$ |
| | | | | RUN | $e_L'$ |
| | | | | C' | $Y$ |
| | | | | RUN | $e_T$ |
| | | | | D' | $Y'$ |
| | | | | RUN | $e_T$ |
| | | | | E' | $S$ |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA: SCOTCHPLY 1002 Gl/Ep NO: 049

PROGRAM LISTING:

```
000 46   045 42   090 05   135 06   180 19
001 11   046 00   091 02   136 81   181 43
002 03   047 03   092 42   137 46   182 09
003 08   048 03   093 06   138 14   183 09
004 93   049 93   094 09   139 43   184 81
005 06   050 03   095 02   140 01   185 55
006 42   051 03   096 00   141 00   186 43
007 00   052 42   097 42   142 81   187 00
008 05   053 00   098 09   143 46   188 06
009 08   054 04   099 08   144 16   189 95
010 93   055 03   100 01   145 43   190 81
011 02   056 03   101 03   146 01   191 46
012 07   057 93   102 08   147 05   192 10
013 42   058 03   103 42   148 81   193 43
014 00   059 42   104 09   149 55   194 00
015 06   060 01   105 09   150 43   195 00
016 93   061 01   106 02   151 00   196 81
017 02   062 05   107 09   152 05   197 55
018 06   063 00   108 42   153 95   198 43
019 42   064 93   109 00   154 81   199 01
020 01   065 02   110 00   155 46   200 00
021 06   066 42   111 00   156 17   201 95
022 04   067 01   112 42   157 43   202 81
023 93   068 02   113 00   158 06
024 01   069 04   114 07   159 09
025 04   070 07   115 42   160 81
026 42   071 93   116 00   161 55
027 01   072 05   117 08   162 43
028 00   073 42   118 42   163 00
029 01   074 01   119 00   164 05
030 03   075 03   120 09   165 95
031 42   076 01   121 43   166 81
032 00   077 00   122 00   167 46
033 01   078 93   123 05   168 18
034 07   079 03   124 81   169 43
035 93   080 42   125 46   170 09
036 04   081 01   126 12   171 08
037 07   082 04   127 43   172 81
038 42   083 07   128 00   173 55
039 00   084 05   129 06   174 43
040 02   085 08   130 81   175 00
041 01   086 42   131 46   176 06
042 05   087 01   132 13   177 95
043 93   088 05   133 43   178 81
044 04   089 05   134 01   179 46
```

TITLE: PLY DATA: NO: 051

| | | | | |
|---|---|---|---|---|
| A' $X, e_L$ | B' $X', e_L'$ | C' $Y, e_T$ | D' $Y', e_T'$ | E' $S, e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

00 $S$

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 $X$ | 16 $X'$ |
| 17 0 | 18 0 | 19 0 | | | |
| 98 $Y$ | 99 $Y'$ | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 051 A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | $X$ |
| | | | | RUN | $e_L$ |
| | | | | B' | $X'$ |
| | | | | RUN | $e_L'$ |
| | | | | C' | $Y$ |
| | | | | RUN | $e_T$ |
| | | | | D' | $Y'$ |
| | | | | RUN | $e_T$ |
| | | | | E' | $S$ |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA NO: 051

PROGRAM LISTING:

TITLE: PLY DATA: NO: 053

| | | | | |
|---|---|---|---|---|
| A' $X, e_L$ | B' $X', e_L'$ | C' $Y, e_T$ | D' $Y', e_T'$ | E' $S, e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $S$ | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 $X$ | 16 $X'$ |
| 17 0 | 18 0 | 19 0 | | | |
| 98 $Y$ | 99 $Y'$ | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 053-- A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | $X$ |
| | | | | RUN | $e_L$ |
| | | | | B' | $X'$ |
| | | | | RUN | $e_L'$ |
| | | | | C' | $Y$ |
| | | | | RUN | $e_T$ |
| | | | | D' | $Y'$ |
| | | | | RUN | $e_T$ |
| | | | | E' | $S$ |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA NO: 053

PROGRAM LISTING:

TITLE: PLY DATA: NO: 055

| | | | | |
|---|---|---|---|---|
| A' X, $e_L$ | B' X', $e_L$' | C' Y, $e_T$ | D' Y', $e_T$' | E' S, $e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 S | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 X | 16 X' |
| 17 0 | 18 0 | 19 0 | | | |
| 98 Y | 99 Y' | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 055 A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | X |
| | | | | RUN | $e_L$ |
| | | | | B' | X' |
| | | | | RUN | $e_L$' |
| | | | | C' | Y |
| | | | | RUN | $e_T$ |
| | | | | D' | Y' |
| | | | | RUN | $e_T$ |
| | | | | E' | S |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA NO: 055

PROGRAM LISTING:

TITLE: PLY DATA: NO: 057

| | | | | |
|---|---|---|---|---|
| A' $X$, $e_L$ | B' $X'$, $e_L'$ | C' $Y$, $e_T$ | D' $Y'$, $e_T'$ | E' $S$, $e_S$ |
| A $E_L$ | B $E_T$ | C $\nu_{LT}$ | D $G_{LT}$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $S$ | | | | | |
| 01 $I_{1Q}$ | 02 $I_{2Q}$ | 03 $R_{1Q}$ | 04 $R_{2Q}$ | 05 $E_L$ | 06 $E_T$ |
| 07 0 | 08 0 | 09 $\nu_{LT}$ | 10 $G_{LT}$ | | |
| 11 $I_{1S}$ | 12 $I_{2S}$ | 13 $R_{1S}$ | 14 $R_{2S}$ | 15 $X$ | 16 $X'$ |
| 17 0 | 18 0 | 19 0 | | | |
| 98 $Y$ | 99 $Y'$ | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 057 A&B | | A | $E_L$ |
| | | | | B | $E_T$ |
| | | | | C | $\nu_{LT}$ |
| | | | | D | $G_{LT}$ |
| | | | | A' | $X$ |
| | | | | RUN | $e_L$ |
| | | | | B' | $X'$ |
| | | | | RUN | $e_L'$ |
| | | | | C' | $Y$ |
| | | | | RUN | $e_T$ |
| | | | | D' | $Y'$ |
| | | | | RUN | $e_T$ |
| | | | | E' | $S$ |
| | | | | RUN | $e_S$ |

PROGRAM TITLE: PLY DATA NO: 057

PROGRAM LISTING:

TITLE: STORAGE OF CONSTITUENT PROPERTIES NO: 059

| | | | | |
|---|---|---|---|---|
| A' $\nu_m$ | B' $\eta_1$ | C' $\eta_2$ | D' $\eta_6$ or $\eta$ | E' $v_f$ |
| A $E_f$, $\alpha_f$ | B $G_f$, $\alpha_m$ | C $\nu_f$ | D $E_m$ | E $G_m$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_f$ | 02 $G_f$ | 03 $\nu_f$ | 04 $E_m$ | 05 $G_m$ | 06 $\nu_m$ |
| 07 $\eta_1$ | 08 $\eta_2$ | 09 $\eta_6$ or $\eta$ | 10 $v_f$ | | |
| 11 $v_m$ | 12 $\alpha_f$ | 13 $\alpha_m$ | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 059 A&B | | INV | |
| | | | | st fg | |
| | | | | 0 | |
| Input data | 2 | | $E_f$ | A | $E_f$ |
| | | | $G_f$ | B | $G_f$ |
| | | | $\nu_f$ | C | $\nu_f$ |
| | | | $E_m$ | D | $E_m$ |
| | | | $G_m$ | E | $G_m$ |
| | | | $\nu_m$ | A' | $\nu_m$ |
| | | | $\eta_1$ | B' | $\eta_1$ |
| | | | $\eta_2$ | C' | $\eta_2$ |
| | | | $\eta_6$ or $\eta$ | D' | $\eta_6$ or $\eta$ |
| | | | $v_f$ | E' | $v_m$ |
| | | | | st flg | |
| | | | | 0 | |
| | | | $\alpha_f$ | A | |
| | | | $\alpha_m$ | B | |

**PROGRAM TITLE:** STORAGE OF CONSTITUENT PROPERTIES **NO:** 059

**PROGRAM LISTING:**

```
000 46   045 46   090 46
001 11   046 14   091 17
002 60   047 60   092 42
003 00   048 00   093 00
004 00   049 00   094 07
005 01   050 05   095 81
006 01   051 06   096 46
007 42   052 42   097 18
008 00   053 00   098 42
009 01   054 04   099 00
010 81   055 81   100 08
011 42   056 42   101 81
012 01   057 01   102 46
013 02   058 05   103 19
014 81   059 81   104 42
015 46   060 46   105 00
016 12   061 15   106 09
017 60   062 60   107 81
018 00   063 00   108 46
019 00   064 00   109 10
020 02   065 07   110 42
021 06   066 01   111 01
022 42   067 42   112 00
023 00   068 00   113 01
024 02   069 05   114 75
025 81   070 81   115 43
026 42   071 42   116 01
027 01   072 01   117 00
028 03   073 06   118 95
029 81   074 81   119 42
030 46   075 46   120 01
031 13   076 16   121 01
032 60   077 60   122 81
033 00   078 00
034 00   079 00
035 04   080 08
036 01   081 06
037 42   082 42
038 00   083 00
039 03   084 06
040 81   085 81
041 42   086 42
042 01   087 01
043 04   088 07
044 81   089 81
```

TITLE: COMPOSITE MODULI I NO: 061

| | | | | |
|---|---|---|---|---|
| A' $E$ | B' $\nu$ | C' | D' | E' |
| A $E_L$ | B $G_{LT}$ | C $\nu_{LT}$ | D $G_{TT}$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_f$ | 02 $G_f$ | 03 $\nu_f$ | 04 $E_m$ | 05 $G_m$ | 06 $\nu_m$ |
| 07 $\eta_1$ | 08 $\eta_2$ | 09 $\eta_6$ or $\eta$ | 10 $v_f$ | | |
| 11 $v_m$ | 12 $\alpha_f$ | 13 $\alpha_m$ | 14 | 15 | 16 |
| 17 used | 18 used | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input data | 1 | 059 A&B | | | |
| Compute moduli | 2 | 061 A&B | | A<br>B<br>C<br>D<br>A'<br>B' | $E_L$<br>$G_{LT}$<br>$\nu_{LT}$<br>$G_{TT}$<br>$E$<br>$\nu$ |

**PROGRAM TITLE:** COMPOSITE MODULI I **NO:** 061

**FORMULAS:**

$$E_1^* = \frac{1}{\eta_1 v_f + v_m} (\eta_1 v_f E_f + v_m E_m) \tag{128}$$

$$\nu_{12}^* = \frac{1}{\eta_1 v_f + v_m} (\eta_1 v_f \nu_f + v_m \nu_m) \tag{129}$$

$$\frac{1}{G_{cc}^*} = \left( \frac{v_f}{G_f} + \frac{\eta_c v_m}{G_m} \right) \frac{1}{v_f + \eta_c v_m} \tag{132}$$

$$\frac{1}{E^*} = \frac{v_f}{E_f} + \frac{\eta v_m}{E_m} \frac{1}{v_f + \eta v_m} \tag{146}$$

$$\nu^* = \frac{v_f \nu_f E_m + \eta v_m \nu_m E_f}{v_f E_m + \eta v_m E_f} \tag{147}$$

**PROGRAM TITLE:** COMPOSITE MODULI I **NO:** 061

**PROGRAM LISTING:**

```
000 46   045 43   090 43   135 00   180 65
001 11   046 01   091 01   136 05   181 43
002 43   047 01   092 08   137 09   182 01
003 00   048 54   093 85   138 46   183 01
004 01   049 95   094 43   139 16   184 65
005 42   050 81   095 01   140 43   185 43
006 01   051 46   096 09   141 00   186 00
007 08   052 12   097 65   142 01   187 06
008 43   053 43   098 43   143 42   188 65
009 00   054 00   099 01   144 01   189 43
010 04   055 09   100 01   145 08   190 00
011 42   056 42   101 55   146 43   191 01
012 01   057 01   102 43   147 00   192 54
013 09   058 09   103 01   148 04   193 55
014 53   059 43   104 07   149 42   194 53
015 43   060 00   105 54   150 01   195 43
016 00   061 02   106 95   151 07   196 01
017 07   062 42   107 81   152 43   197 00
018 65   063 01   108 46   153 00   198 65
019 43   064 08   109 13   154 09   199 43
020 01   065 43   110 43   155 42   200 00
021 00   066 00   111 00   156 01   201 04
022 65   067 05   112 03   157 09   202 85
023 43   068 42   113 42   158 41   203 43
024 01   069 01   114 01   159 00   204 00
025 08   070 07   115 08   160 07   205 09
026 85   071 53   116 43   161 01   206 65
027 43   072 43   117 00   162 46   207 43
028 01   073 01   118 06   163 17   208 01
029 01   074 00   119 42   164 53   209 01
030 65   075 85   120 01   165 43   210 65
031 43   076 43   121 09   166 01   211 43
032 01   077 01   122 41   167 00   212 00
033 09   078 09   123 00   168 65   213 01
034 54   079 65   124 01   169 43   214 54
035 55   080 43   125 04   170 00   215 95
036 53   081 01   126 46   171 03   216 81
037 43   082 01   127 14   172 65
038 00   083 54   128 43   173 43
039 07   084 55   129 00   174 00
040 65   085 53   130 08   175 04
041 43   086 43   131 42   176 85
042 01   087 01   132 01   177 43
043 00   088 00   133 09   178 00
044 85   089 55   134 41   179 09
```

## SAMPLE PROBLEM: 061 COMPOSITE MODULI I NO: 061

Input data with Program 059

$E_f = 10.5 \times 10^6$

$G_f = 4.38 \times 10^6$

$\nu_f = .2$

$E_m = .5 \times 10^6$

$G_m = .185 \times 10^6$

$\nu_m = .35$

$\eta_1 = 1$

$\eta_2 = .5$

$\eta_6$ or $\eta = .4$

$v_f = .6$

$E_L = 6.5 \times 10^6$

$G_{LT} = 758.6 \times 10^3$

$\nu_{LT} = .25$

$G_{TT} = 656.8 \times 10^6$

$E = 2.0152 \times 10^6$

$\nu = .3273$

TITLE: COMPOSITE MODULI II NO: 065

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $E_T$ | B $\nu_{TT}$ | C | D | E |

REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_f$ | 02 $G_f$ | 03 $\nu_f$ | 04 $E_m$ | 05 $G_m$ | 06 $\nu_m$ |
| 07 $\eta_1$ | 08 $\eta_2$ | 09 $\eta_6$ or $\eta$ | 10 $v_f$ | | |
| 11 $v_m$ | 12 $\alpha_f$ | 13 $\alpha_m$ | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Store data with Program 059 | 1 | | | | |
| Compute moduli | 2 | 065 A&B | <br>$G_{TT}$ | A<br>RUN | $E_T$<br>$\nu_{TT}$ |

**PROGRAM TITLE:** COMPOSITE MODULI II **NO:** 065

**FORMULAS:**

$$\frac{1}{E_a^*} = \left(\frac{v_f}{E_f} + \frac{\eta_a v_m}{E_m}\right) \frac{1}{v_f + \eta_a v_m} - v_f v_m \frac{(\eta_1 \eta_a E_f \nu_m - E_m \nu_f)(\nu_m / E_m - \nu_f / E_f)}{(\eta_1 v_f E_f + v_m E_m)(v_f + \eta_a v_m)} \tag{130}$$

$$\nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

**PROGRAM TITLE:** COMPOSITE MODULI II **NO:** 065

**PROGRAM LISTING:**

```
000 46   045 65   090 53   135 55
001 11   046 53   091 53   136 43
002 53   047 43   092 43   137 01
003 43   048 00   093 00   138 09
004 01   049 07   094 07   139 54
005 00   050 65   095 65   140 20
006 55   051 43   096 43   141 75
007 43   052 00   097 01   142 01
008 00   053 08   098 00   143 95
009 01   054 65   099 65   144 81
010 85   055 43   100 43
011 43   056 00   101 00
012 00   057 01   102 01
013 08   058 65   103 85
014 65   059 43   104 43
015 43   060 00   105 01
016 01   061 06   106 01
017 01   062 75   107 65
018 55   063 43   108 43
019 43   064 00   109 00
020 00   065 04   110 04
021 04   066 65   111 54
022 54   067 43   112 65
023 55   068 00   113 53
024 53   069 03   114 43
025 43   070 54   115 01
026 01   071 65   116 00
027 00   072 53   117 85
028 85   073 43   118 43
029 43   074 00   119 00
030 00   075 06   120 08
031 08   076 55   121 65
032 65   077 43   122 43
033 43   078 00   123 01
034 01   079 04   124 01
035 01   080 75   125 54
036 54   081 43   126 54
037 75   082 00   127 95
038 43   083 03   128 20
039 01   084 55   129 42
040 00   085 43   130 01
041 65   086 00   131 09
042 43   087 01   132 81
043 01   088 54   133 65
044 01   089 55   134 02
```

## SAMPLE PROBLEM: 065 COMPOSITE MODULI NO: 065

Input data with Program 059

$E_f = 10.5 \times 10^6$ $E_m = .5 \times 10^6$

$G_f = 4.38 \times 10^6$ $G_m = .185 \times 10^6$

$\nu_f = .2$ $\nu_m = .35$

$\eta_1 = 1$

$\eta_2 = .5$ $G_{TT} = 656.8 \times 10^3$

$\eta_6$ or $\eta = .4$

$V_f = .6$

$E_T = 1.9349 \times 10^6$

$\nu = .473$

TITLE: THERMAL EXPANSION COEFFICIENTS NO: 069

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $\alpha_L$ | B $\alpha_T$ | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_f$ | 02 $G_f$ | 03 $\nu_f$ | 04 $E_m$ | 05 $G_m$ | 06 $\nu_m$ |
| 07 $\eta_1$ | 08 $\eta_2$ | 09 $\eta_6$ or $\eta$ | 10 $v_f$ | | |
| 11 $v_m$ | 12 $\alpha_f$ | 13 $\alpha_m$ | 14 | 15 | 16 |
| 17 | 18 used | 19 used | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Store data with Program 059 | 1 | | | | |
| Compute expansion | 2 | 069 A&B | | A<br>B | $\alpha_L$<br>$\alpha_T$ |

## PROGRAM TITLE: THERMAL EXPANSION COEFFICIENTS NO: 069

## FORMULAS:

$$\alpha_L = \alpha_1^* = (v_f E_f \alpha_f + v_m E_m \alpha_m)/(\eta_1 v_f E_f + v_m E_m) \quad (162)$$

$$\alpha_T = \alpha_2^* = v_f \alpha_f + v_m \alpha_m + (v_f \alpha_f \nu_f + v_m \alpha_m \nu_m) - (\eta_1 v_f \nu_f + v_m \nu_m)\alpha_1^* \quad (163)$$

## PROGRAM LISTING:

P

```
000 46    030 00    060 00    090 00
001 11    031 07    061 03    091 07
002 53    032 65    062 54    092 65
003 43    033 43    063 65    093 43
004 01    034 01    064 43    094 00
005 00    035 00    065 01    095 03
006 65    036 65    066 00    096 85
007 43    037 43    067 65    097 43
008 00    038 00    068 43    098 00
009 01    039 01    069 01    099 06
010 65    040 85    070 02    100 54
011 43    041 43    071 85    101 65
012 01    042 01    072 53    102 43
013 02    043 01    073 01    103 01
014 85    044 65    074 85    104 01
015 43    045 43    075 43    105 65
016 01    046 00    076 00    106 43
017 01    047 04    077 06    107 01
018 65    048 54    078 54    108 09
019 43    049 95    079 85    109 95
020 00    050 42    080 43    110 81
021 04    051 01    081 01
022 65    052 09    082 01
023 43    053 81    083 65
024 01    054 46    084 43
025 03    055 12    085 01
026 54    056 53    086 03
027 55    057 01    087 75
028 53    058 85    088 53
029 43    059 43    089 43
```

TITLE: LONGITUDINAL STRENGTH NO: 071

| | | | | |
|---|---|---|---|---|
| A' X (brittle) | B' X (ductile) | C' $\delta$ | D' X (Statist) | E' |
| A $X_f$ | B $X_m$ | C $\delta_f$ | D $X_{fo}$ | E $\alpha$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_f$ | 02 $G_f$ | 03 $\nu_f$ | 04 $E_m$ | 05 $G_m$ | 06 $\nu_m$ |
| 07 | 08 | 09 | 10 $v_f$ | | |
| 11 $v_m$ | 12 | 13 | 14 $X_f$ | 15 $X_m$ | 16 $\delta_f$ |
| 17 $X_{fo}$ | 18 $\alpha$ | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

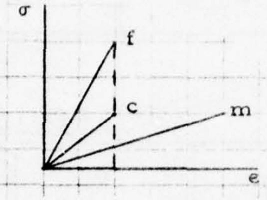
| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Store data with Program 059 | 1 | | | | |
| Input data | 2 | 071 A&B | $X_f$<br>$X_m$<br>$\delta_f$<br>$X_{fo}$<br>$\alpha$ | A<br>B<br>C<br>D<br>E | $X_f$<br>$X_m$<br>$\delta_f$<br>$X_{fo}$<br>$\alpha$ |
| Compute strength | 3 | | | A'<br>B'<br>C'<br>RUN | X(Brittle)<br>X(Ductile)<br>$\delta$<br>X(Statistical) |

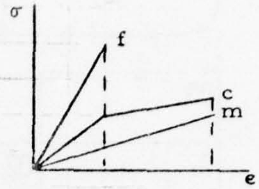
**PROGRAM TITLE:** LONGITUDINAL STRENGTH **NO:** 071

**FORMULAS:**

Brittle matrix

$$X = [v_f + (1-v_f)E_m/E_f]X_f \qquad (164)$$





Ductile matrix (tough)

$$X = v_f X_f + v_m X_m \qquad (165)$$

$$\bar{\delta} = \frac{\delta}{d_f} = \left[\frac{1}{2}\,\frac{E_f}{G_m}\,\frac{1-\sqrt{v_f}}{\sqrt{v_f}}\right]^{1/2} \qquad (188)$$

$$X = v_f \text{ (strength of bundle of length } \delta)$$

$$= v_f X_B (L/\delta)^{-1/\alpha} \qquad (182)$$

$$= v_f X_{fo} (\delta \alpha e)^{-1/\alpha} \qquad (183)$$

**PROGRAM TITLE:** LONGITUDINAL STRENGTH **NO:** 071

**PROGRAM LISTING:**

```
000 46   045 43   090 43
001 11   046 00   091 00
002 42   047 01   092 01
003 01   048 54   093 55
004 04   049 65   094 43
005 81   050 43   095 00
006 46   051 01   096 05
007 12   052 04   097 55
008 42   053 95   098 02
009 01   054 81   099 95
010 05   055 46   100 30
011 81   056 17   101 65
012 46   057 43   102 43
013 13   058 01   103 01
014 42   059 00   104 06
015 01   060 65   105 95
016 06   061 43   106 81
017 81   062 01   107 65
018 46   063 04   108 43
019 14   064 85   109 01
020 42   065 43   110 08
021 01   066 01   111 65
022 07   067 01   112 01
023 81   068 65   113 22
024 46   069 43   114 23
025 15   070 01   115 95
026 42   071 05   116 45
027 01   072 95   117 43
028 08   073 81   118 01
029 81   074 46   119 08
030 46   075 18   120 20
031 16   076 53   121 94
032 53   077 01   122 95
033 43   078 75   123 65
034 01   079 43   124 43
035 00   080 01   125 01
036 85   081 00   126 07
037 43   082 30   127 65
038 01   083 54   128 43
039 01   084 55   129 01
040 65   085 43   130 00
041 43   086 01   131 95
042 00   087 00   132 81
043 04   088 30
044 55   089 65
```

TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ NO: 075

| | | | | |
|---|---|---|---|---|
| A' | B' $\delta_{1A}$ | C' $\delta_{2A}$ | D' | E' |
| A Initialize | B $I_{1A}$ | C $I_{2A}$ | D $R_{1A}$ | E $R_{2A}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $I_1 = I_{1A}$ | 02 $I_2 = I_{2A}$ | 03 $R_1$, $R_{1A}$ | 04 $R_2$, $R_{2A}$ | 05 | 06 |
| 07 $\delta_1$ | 08 $\delta_2$ | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 $n_0$ | 18 $n_{90}$ | 19 $n$ | | | |
| 98 $n_{45}$ | 99 $n_{-45}$ | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input ply data and initialize | 1 | 045 A&B | | A | $E_L$ |
| Input $n_\theta$ and | 2 | 075 A&B | N | A | N |
| Compute invariants of $A_{ij}$ | | | $n_\theta$ | RUN | $n_0/N$ |
| | | | $n_{90}$ | RUN | $n_{90}/N$ |
| | | | $n_{45}$ | RUN | $n_{45}/N$ |
| | | | $n_{-45}$ | RUN | $n_{-45}/N$ |
| | | | | B | $I_{1A}$ |
| | | | | C | $I_{2A}$ |
| | | | | D | $R_{1A}$ |
| | | | | E | $R_{2A}$ |
| | | | | B' | $\delta_{1A}$ |
| | | | | C' | $\delta_{2A}$ |

PROGRAM TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ NO: 075

FORMULAS:

TABLE 29 FORMULAS FOR IN-PLANE MODULUS FOR $[0_p/90_q/45_r/-45_s]$

| | $I_1$ | $I_2$ | $\sqrt{V_1^2 + V_3^2}\ R_1$ | $\sqrt{V_2^2 + V_4^2}\ R_2$ |
|---|---|---|---|---|
| $A'_{11}/h$ | 1 | 1 | $\cos 2(\theta - \delta_1)$ | $\cos 4\theta$ |
| $A'_{22}/h$ | 1 | 1 | $-\cos 2(\theta - \delta_1)$ | $\cos 4\theta$ |
| $A'_{12}/h$ | 1 | -1 | 0 | $-\cos 4\theta$ |
| $A'_{66}/h$ | 0 | 1 | 0 | $-\cos 4\theta$ |
| $A'_{16}/h$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta - \delta_1)$ | $-\sin 4\theta$ |
| $A'_{26}/h$ | 0 0 | 0 | $-\frac{1}{2}\sin 2(\theta - \delta_1)$ | $\sin 4\theta$ |

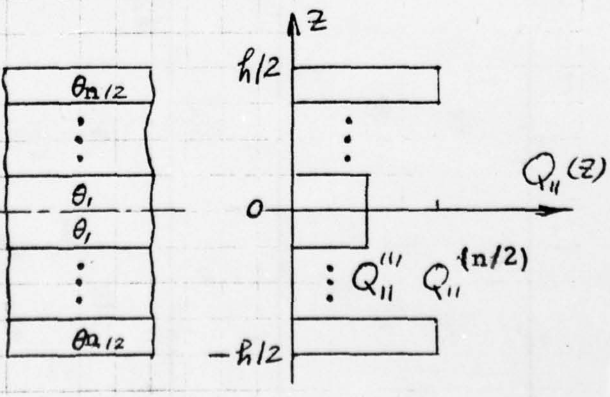
$$\sqrt{V_1^2 + V_3^2} = \frac{1}{N}\sqrt{(n_o - n_{90})^2 + (n_{45} - n_{-45})^2} = R_{1A} / R_1 \qquad (229)$$

$$\sqrt{V_2^2 + V_4^2} = \frac{1}{N}\left|(n_o + n_{90} - n_{45} - n_{-45})\right| = R_{2A} / R_2 \qquad (230)$$

$$\tan 2\delta_1 = -\frac{n_{45} - n_{-45}}{n_o - n_{90}} \qquad (231)$$

Laminate is orthotropic if $n_{45} = n_{-45}$.

Figure 61 Ply orientations and modulus of symmetric laminates.



**PROGRAM TITLE:** IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ **NO:** 075

**PROGRAM LISTING:**

```
000 46   045 43   090 49   135 81   180 07
001 11   046 09   091 00   136 46   181 81
002 42   047 08   092 04   137 87   182 46
003 01   048 95   093 43   138 43   183 18
004 09   049 40   094 09   139 01   184 43
005 81   050 85   095 08   140 06   185 00
006 55   051 53   096 75   141 90   186 08
007 43   052 43   097 43   142 88   187 81
008 01   053 01   098 09   143 04
009 09   054 07   099 09   144 05
010 95   055 75   100 95   145 42
011 42   056 43   101 42   146 00
012 01   057 01   102 01   147 07
013 07   058 08   103 06   148 41
014 81   059 54   104 55   149 89
015 55   060 40   105 53   150 46
016 43   061 95   106 43   151 88
017 01   062 30   107 01   152 00
018 09   063 42   108 07   153 42
019 95   064 00   109 75   154 00
020 42   065 05   110 43   155 07
021 01   066 49   111 01   156 41
022 08   067 00   112 08   157 89
023 81   068 03   113 54   158 46
024 55   069 43   114 90   159 13
025 43   070 01   115 87   160 43
026 01   071 07   116 95   161 00
027 09   072 85   117 94   162 02
028 95   073 43   118 22   163 81
029 42   074 01   119 34   164 46
030 09   075 08   120 55   165 14
031 08   076 75   121 02   166 43
032 81   077 43   122 95   167 00
033 55   078 09   123 42   168 03
034 43   079 08   124 00   169 81
035 01   080 75   125 07   170 46
036 09   081 43   126 46   171 15
037 95   082 09   127 89   172 43
038 42   083 09   128 00   173 00
039 09   084 95   129 42   174 04
040 09   085 40   130 00   175 81
041 81   086 30   131 08   176 46
042 46   087 42   132 43   177 17
043 12   088 00   133 00   178 43
044 75   089 06   134 01   179 00
```

NO: 075

SAMPLE PROBLEM: 075 IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/45_s]$

1. T-300/5208

$I_1 = 49.49$, $I_2 = 26.88$, $R_1 = 85.732$, $R_2 = 19.717$

a) [0/90] $I_{1A} = 49.49$ $R_{1A} = 0$ $\delta_{1A} = 0 = \delta_2$

$I_{2A} = 26.88$ $R_{2A} = 19.72$

b) [45/-45] $I_{1A} = 49.49$ $R_{1A} = 0$ $\delta_{1A} = 0 = \delta_2$

$I_{2A} = 26.88$ $R_{2A} = 19.72$

c) [0/90/45/-45] $I_{1A} = 49.49$ $R_{1A} = 0$ $\delta_{1A} = 0 = \delta_2$

$I_{2A} = 26.88$ $R_{2A} = 0$

d) [0/45] $I_{1A} = 49.49$ $R_{1A} = 60.62$ $\delta_{1A} = -22.50$

$I_{2A} = 26.88$ $R_{2A} = 0$ $\delta_{2A} = 0$

2. Same as 1 but for B(4)/5505

a) [0/90] $I_{1A} = 58.0$ $R_{1A} = 0$ $\delta_1 = 0 = \delta_2$

$I_{2A} = 29.80$ $R_{2A} = 24.0$

TITLE: IN-PLANE MODULUS AND COMPLIANCE NO: 079

| | | | | |
|---|---|---|---|---|
| A' | B' $\frac{A}{h}_{16}$, $ha_{16}$ | C' $\frac{A}{h}_{26}$, $ha_{26}$ | D' | E' |
| A $\frac{1}{h}\lvert A_{ij}\rvert$ | B $\frac{A}{h}_{11}$, $ha_{11}$ | C $\frac{A}{h}_{22}$, $ha_{22}$ | D $\frac{A}{h}_{12}$, $ha_{12}$ | E $\frac{A}{h}_{66}$, $ha_{66}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $\theta$ | | | | | |
| 01 $I_{1A}$, $ha_{11}$ | 02 $I_{2A}$, $ha_{22}$ | 03 $R_{1A}$, $ha_{12}$ | 04 $R_{2A}$, $ha_{66}$ | 05 $ha_{16}$ | 06 $ha_{26}$ |
| 07 $\delta_{1A}$ | 08 $\delta_{2A}$ | 09 | 10 | | |
| 11 $\frac{A}{h}_{11}$ | 12 $\frac{A}{h}_{22}$ | 13 $\frac{A}{h}_{12}$ | 14 $\frac{A}{h}_{66}$ | 15 $\frac{A}{h}_{16}$ | 16 $\frac{A}{h}_{26}$ |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from 075 | | | | | |
| Compute $A_{ij}$ | 3 | 041 A&B | $\theta$ (deg) | B | $A_{11}/h$ |
| | | | | C | $A_{22}/h$ |
| | | | | D | $A_{12}/h$ |
| | | | | E | $A_{66}/h$ |
| | | | | B' | $A_{16}/h$ |
| | | | | C' | $A_{26}/h$ |
| Invert $A_{ij}$ | 4 | 027 A&B | | A | $ha_{26}$ |
| | | | | RCL 01 | $ha_{11}$ |
| | | | | RCL 02 | $ha_{22}$ |
| | | | | RCL 03 | $ha_{12}$ |
| | | | | RCL 04 | $ha_{66}$ |
| | | | | RCL 05 | $ha_{16}$ |
| | | | | RCL 06 | $ha_{26}$ |

PROGRAM TITLE: IN-PLANE MODULUS AND COMPLIANCE NO: 079

FORMULAS:

TABLE 29 FORMULAS FOR IN-PLANE MODULUS FOR $[0_p/90_q/45_r/-45_s]$

|  | $I_1$ | $I_2$ | $R_{1A}$ | $R_{2A}$ |
|---|---|---|---|---|
| $A'_{11}/h$ | 1 | 1 | $\cos 2(\theta-\delta_1)$ | $\cos 4\theta$ |
| $A'_{22}/h$ | 1 | 1 | $-\cos 2(\theta-\delta_1)$ | $\cos 4\theta$ |
| $A'_{12}/h$ | 1 | -1 | 0 | $-\cos 4\theta$ |
| $A'_{66}/h$ | 0 | 1 | 0 | $-\cos 4\theta$ |
| $A'_{16}/h$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $-\sin 4\theta$ |
| $A'_{26}/h$ | 0 0 | 0 | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $\sin 4\theta$ |

Let Compliance $= a_{ij} = [A_{ij}]^{-1}$ or $a_{ij}A_{jk} = \delta_{ik}$ (204)

where

$$a_{11} = \frac{1}{\Delta}(A_{22}A_{66}-A_{26}^2), \quad a_{22} = \frac{1}{\Delta}(A_{11}A_{66}-A_{16}^2)$$

$$a_{12} = \frac{1}{\Delta}(A_{16}A_{26}-A_{12}A_{66}), \quad a_{66} = \frac{1}{\Delta}(A_{11}A_{22}-A_{12}^2) \tag{206}$$

$$a_{16} = \frac{1}{\Delta}(A_{12}A_{26}-A_{22}A_{16}), \quad a_{26} = \frac{1}{\Delta}(A_{12}A_{16}-A_{11}A_{26})$$

$$\Delta = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}$$

In-Plane Engineering Constants:

$$E_{11}^o = \frac{1}{ha_{11}}, \quad E_{22}^o = \frac{1}{ha_{22}}, \quad \nu_{12}^o = -\frac{a_{12}}{a_{11}}, \quad \nu_{21}^o = -\frac{a_{12}}{a_{22}}, \quad G_{12}^o = \frac{1}{ha_{66}}$$

| TITLE: IN-PLANE STRAIN | | | | NO: 081 |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $N_i$ | B $\epsilon_1^o$ | C $\epsilon_2^o$ | D $\epsilon_6^o$ | E |

| 00 | | | | | |
|---|---|---|---|---|---|

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 01 $a_{11}$ | 02 $a_{22}$ | 03 $a_{12}$ | 04 $a_{66}$ | 05 $a_{16}$ | 06 $a_{26}$ |
| 07 $N_1$ | 08 $N_2$ | 09 $N_6$ | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 $\epsilon_1^o$ | 18 $\epsilon_2^o$ | 19 $\epsilon_6^o$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from 079 | | | | | |
| Input $N_i$ | 5 | 019 A&B | $N_1$ | A | $N_1$ |
| | | | $N_2$ | RUN | $N_2$ |
| | | | $N_6$ | RUN | $N_6$ |
| Compute $\epsilon_i^o$ | | | | B | $\epsilon_1^o$ |
| | | | | C | $\epsilon_2^o$ |
| | | | | D | $\epsilon_6^o$ |

PROGRAM TITLE: IN-PLANE STRAIN NO: 081

FORMULAS:

$$\text{Let Compliance} = a_{ij} = [A_{ij}]^{-1} \text{ or } a_{ij}A_{jk} = \delta_{ik} \tag{204}$$

$$\begin{Bmatrix} \epsilon_1^o \\ \epsilon_2^o \\ \epsilon_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \text{ or } \epsilon_i^o = a_{ij}N_j \tag{205}$$

TITLE: PLY STRESS FOR $[0_p/90_q/45_r/-45_s]$ NO: 083

| | | | | |
|---|---|---|---|---|
| A' | B' $Q_{16}$ | C' $Q_{26}$ | D' | E' |
| A | B $Q_{11}, \sigma_1$ | C $Q_{22}, \sigma_2$ | D $Q_{12}, \sigma_6$ | E $Q_{66}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $\alpha_i$ | | | | | |
| 01 $I_1$ | 02 $I_2$ | 03 $R_1$ | 04 $R_2$ | 05 | 06 |
| 07 $\sigma_1$ | 08 $\sigma_2$ | 09 $\sigma_6$ | 10 | | |
| 11 $Q_{11}$ | 12 $Q_{22}$ | 13 $Q_{12}$ | 14 $Q_{66}$ | 15 $Q_{16}$ | 16 $Q_{26}$ |
| 17 $e_1^o$ | 18 $e_2^o$ | 19 $e_6^o$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from 081 | | | | | |
| Input ply data and initialize | 6 | 045 A&B | | A | $E_L$ |
| Compute $Q_{ij}(\alpha)$ | 7 | 041 A&B | $\alpha_1$(deg) | B | $Q_{11}$ |
| Compute $\sigma_i^{(\alpha)}$ | 8 | 023 A&B | | B | $\sigma_1$ |
| | | | | C | $\sigma_2$ |
| | | | | D | $\sigma_6$ |
| Compute $Q_{ij}(\beta)$ | Rpt 7 | 041 A&B | $\alpha_i$ (deg)<br>$(i=2, 3, \cdots n)$ | B | $Q_{11}$ |
| Compute $\sigma_i^{(\beta)}$ | Rpt 8 | 023 A&B | | B | $\sigma_1$ |
| | | | | C | $\sigma_2$ |
| | | | | D | $\sigma_6$ |
| Repeat steps | 7 and 8 | for each successive ply. | | | |

## PROGRAM TITLE: PLY STRESS FOR $[0_p/90_q/45_r/-45_s]$ NO: 083

## FORMULAS:

a. Calculation of In-Plane Strains

$$\begin{aligned} e_1^o &= a_{11}N_1 + a_{12}N_2 + a_{16}N_6 \\ e_2^o &= a_{21}N_1 + a_{22}N_2 + a_{26}N_6 \\ e_6^o &= a_{61}N_1 + a_{62}N_2 + a_{66}N_6 \end{aligned} \tag{233}$$

or

$$\begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or} \quad e_i^o = a_{ij}N_j \tag{234}$$

b. Calculation of Stresses of the t-th ply.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}^{(t)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}^{(t)} \begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} \tag{235}$$

For the a oriented ply, replace index "t" by "a" in the equation above

c. TABLE 31 SAMPLE CALCULATIONS OF PLY STRESSES

(For T-300/5208)

| Laminate | Imposed $N_i$ | Resulting $\epsilon_i^o$ | $\sigma_i^{(o)}$ | $\sigma_i^{(90)}$ | $\sigma_i^{(45)}$ | $\sigma_i^{(-45)}$ |
|---|---|---|---|---|---|---|
| [0, 90] | 1 | .0104 | 1.893 | .1088 | | |
| | 0 | -.0003 | .0269 | .0269 | | |
| | 0 | 0 | 0 | .0282 | | |
| [0/90] | 1 | .0101 | 1.8662 | -.2279 | | |
| | 1 | .0101 | .1338 | 1.4495 | | |
| | 1 | .1395 | 1.0 | .9432 | | |
| [45/-45] | 1 | .0104 | | | .5769 | .5591 |
| | 0 | -.003 | | | .4231 | .4224 |
| | 0 | 0 | | | -.4331 | .4268 |
| [0/90/45/-45] | 1 | .0144 | 2.5971 | .1412 | .6399 | .6060 |
| | 0 | -.0042 | -.0024 | -.7294 | .3679 | .3584 |
| | 0 | 0 | 0 | -.0393 | -.4144 | .4236 |

| TITLE: INVARIANTS OF FLEXURAL RIGIDITY | | | | NO: 085 |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A Initialize | B | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $2\alpha_t$ | | | | | |
| 01 $I_1$ , $I_{1D}$ | 02 $I_2$ , $I_{2D}$ | 03 $R_1, R_{1D}$ | 04 $R_2, R_{2D}$ | 05 t | 06 $3t^2-3t+1$ |
| 07 $\delta_{1D}$ | 08 $\delta_{2D}$ | 09 | 10 | | |
| 11 Fc2D | 12 Fs2D | 13 Fc4D | 14 Fs4D | 15 $2z_o/h$ | 16 |
| 17 | 18 | 19 $4\alpha_t$ | | | |
| 98 | 99 n/2 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input ply data | 1 | 045 A&B | | A | $E_L$ |
| Input data | 2. | | N/2 | STO 99 | N/2 |
| | | | $2z_o/h$ | STO 15 | $2z_o/h$ |
| Initialize | 3 | 085 A&B | | A | $F_t(1)$ |
| Compute invariants of $D_{ij}$ | | | $\alpha_1$ (deg) | B | $F_t(2)$ |
| | | | $\alpha_2$ (deg) | RUN | $F_t(3)$ |
| | | | . | . | . |
| | | | . | . | . |
| | | | . | . | . |
| | | | $\alpha_{n/2}$ | RUN | $\frac{R_{1D}}{R_1}$ |
| Recall | | | | RCL 01 | $I_{1D}$ |
| | | | | RCL 02 | $I_{2D}$ |
| | | | | RCL 03 | $R_{1D}$ |
| | | | | RCL 04 | $R_{2D}$ |
| | | | | RCL 07 | $\delta_1$ |
| | | | | RCL 08 | $\delta_2$ |

**PROGRAM TITLE:** INVARIANTS OF FLEXURAL RIGIDITY **NO:** 085

**FORMULAS:**

TABLE 33 FORMULAS FOR FLEXURAL RIGIDITY WITH ISOTROPIC SUBSTRUCTURE

| | $\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_1$ | $\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_2$ | $\sqrt{V_1^2+V_3^2}\ R$ | $\sqrt{V_2^2+V_4^2}\ R_2$ |
|---|---|---|---|---|
| $\frac{12}{h^3}D'_{11}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$ | 1 | 1 | $\cos 2(\theta-\delta_1)$ | $\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{22}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$ | 1 | 1 | $-\cos 2(\theta-\delta_1)$ | $\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{12}-\left(\frac{2z_o}{h}\right)^3 Q^o_{12}$ | 1 | -1 | 0 | $-\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{66}-\frac{1}{2}\left(\frac{2z_o}{h}\right)^3 (Q^o_{11}-Q^o_{12})$ | 0 | 1 | 0 | $-\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{16}$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $-\sin 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{26}$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $\sin 4(\theta-\delta_2)$ |

$$\sqrt{V_1^2 + V_3^2} = \left(\frac{2}{N}\right)^3 \sqrt{\left(\sum F_t \cos 2\alpha_t\right)^2 + \left(\sum F_t \sin 2\alpha_t\right)^2} = R_{1D} / R_1 \qquad (259)$$

$$\sqrt{V_2^2 + V_4^2} = \left(\frac{2}{N}\right)^3 \sqrt{\left(\sum F_t \cos 4\alpha_t\right)^2 + \left(\sum F_t \sin 4\alpha_t\right)^2} = R_{2D} / R_2 \qquad (260)$$

$$\tan 2\delta_2 = \frac{2(D_{16} + D_{26})}{D_{11} - D_{22}} = -\frac{V_3}{V_1} \qquad (261)$$

$$\tan 4\delta_2 = \frac{4(D_{16} - D_{26})}{D_{11} + D_{22} - 2D_{12} - 4D_{66}} = -\frac{V_4}{V_2} \qquad (262)$$

$$F_t = 3\left(t + \frac{z_o}{h_o}\right)^2 - 3\left(t + \frac{z_o}{h_o}\right) + 1 \qquad (263)$$

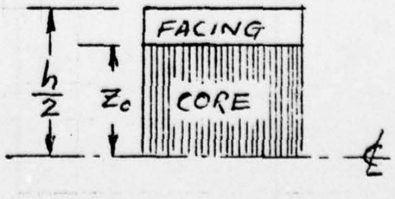


Figure 70 Sandwich Plate

**PROGRAM TITLE:** INVARIANTS OF FLEXURAL RIGIDITY **NO:** 085

**PROGRAM LISTING:**

```
000 46   045 65   090 00   135 05   180 40
001 11   046 03   091 06   136 85   181 85
002 01   047 75   092 95   137 01   182 43
003 75   048 43   093 44   138 95   183 01
004 43   049 00   094 01   139 41   184 04
005 01   050 05   095 03   140 88   185 40
006 05   051 65   096 43   141 46   186 95
007 45   052 03   097 00   142 87   187 30
008 03   053 85   098 00   143 43   188 55
009 95   054 01   099 32   144 01   189 43
010 49   055 95   100 65   145 02   190 09
011 00   056 42   101 43   146 55   191 09
012 01   057 00   102 00   147 43   192 45
013 49   058 06   103 06   148 01   193 03
014 00   059 81   104 95   149 01   194 95
015 02   060 46   105 44   150 94   195 49
016 00   061 12   106 01   151 95   196 00
017 42   062 65   107 02   152 22   197 04
018 01   063 04   108 43   153 34   198 43
019 01   064 94   109 01   154 55   199 01
020 42   065 95   110 09   155 02   200 01
021 01   066 42   111 32   156 95   201 40
022 02   067 01   112 65   157 42   202 85
023 42   068 09   113 43   158 00   203 43
024 01   069 55   114 00   159 07   204 01
025 03   070 02   115 06   160 43   205 02
026 42   071 95   116 95   161 01   206 40
027 01   072 42   117 44   162 04   207 95
028 04   073 00   118 01   163 55   208 30
029 01   074 00   119 04   164 43   209 55
030 85   075 33   120 43   165 01   210 43
031 43   076 65   121 09   166 03   211 09
032 01   077 43   122 09   167 94   212 09
033 05   078 00   123 75   168 95   213 45
034 65   079 06   124 43   169 22   214 03
035 43   080 95   125 00   170 34   215 95
036 09   081 44   126 05   171 55   216 49
037 09   082 01   127 95   172 04   217 00
038 95   083 01   128 90   173 95   218 03
039 46   084 43   129 87   174 42   219 81
040 88   085 01   130 22   175 00
041 42   086 09   131 80   176 08
042 00   087 33   132 87   177 43
043 05   088 65   133 43   178 01
044 40   089 43   134 00   179 03
```

TITLE: FLEXURAL RIGIDITY AND COMPLIANCE NO: 088

| | | | | |
|---|---|---|---|---|
| A' | B' $D_{16}, d_{16}$ | C' $D_{26}, d_{26}$ | D' | E' |
| A $\lvert D_{ij} \rvert$ | B $D_{11}, d_{11}$ | C $D_{22}, d_{22}$ | D $D_{12}, d_{12}$ | E $D_{66}, d_{66}$ |

| | | | | | |
|---|---|---|---|---|---|
| 00 $\theta$ | | | | | |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 01 $I_1, d_{11}$ | 02 $I_2, d_{22}$ | 03 $R_1, d_{12}$ | 04 $R_2, d_{66}$ | 05 $d_{16}$ | 06 $d_{26}$ |
| 07 $\delta_1$ | 08 $\delta_2$ | 09 | 10 | | |
| 11 $D_{11}$ | 12 $D_{22}$ | 13 $D_{12}$ | 14 $D_{66}$ | 15 $D_{16}$ | 16 $D_{26}$ |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from 085 | | | | | |
| Compute $D_{ij}$ | 4 | 041, A&B | $\theta$(deg) | B | $D_{11}$ |
| | | | | C | $D_{22}$ |
| | | | | D | $D_{12}$ |
| | | | | E | $D_{66}$ |
| | | | | B' | $D_{16}$ |
| | | | | C' | $D_{26}$ |
| Invert $D_{ij}$ | 5 | 027 A&B | | A | $d_{26}$ |
| | | | | RCL 01 | $d_{11}$ |
| | | | | RCL 02 | $d_{22}$ |
| | | | | RCL 03 | $d_{12}$ |
| | | | | RCL 04 | $d_{66}$ |
| | | | | RCL 05 | $d_{16}$ |
| | | | | RCL 06 | $d_{26}$ |

PROGRAM TITLE: FLEXURAL RIGIDITY AND COMPLIANCE NO: 088

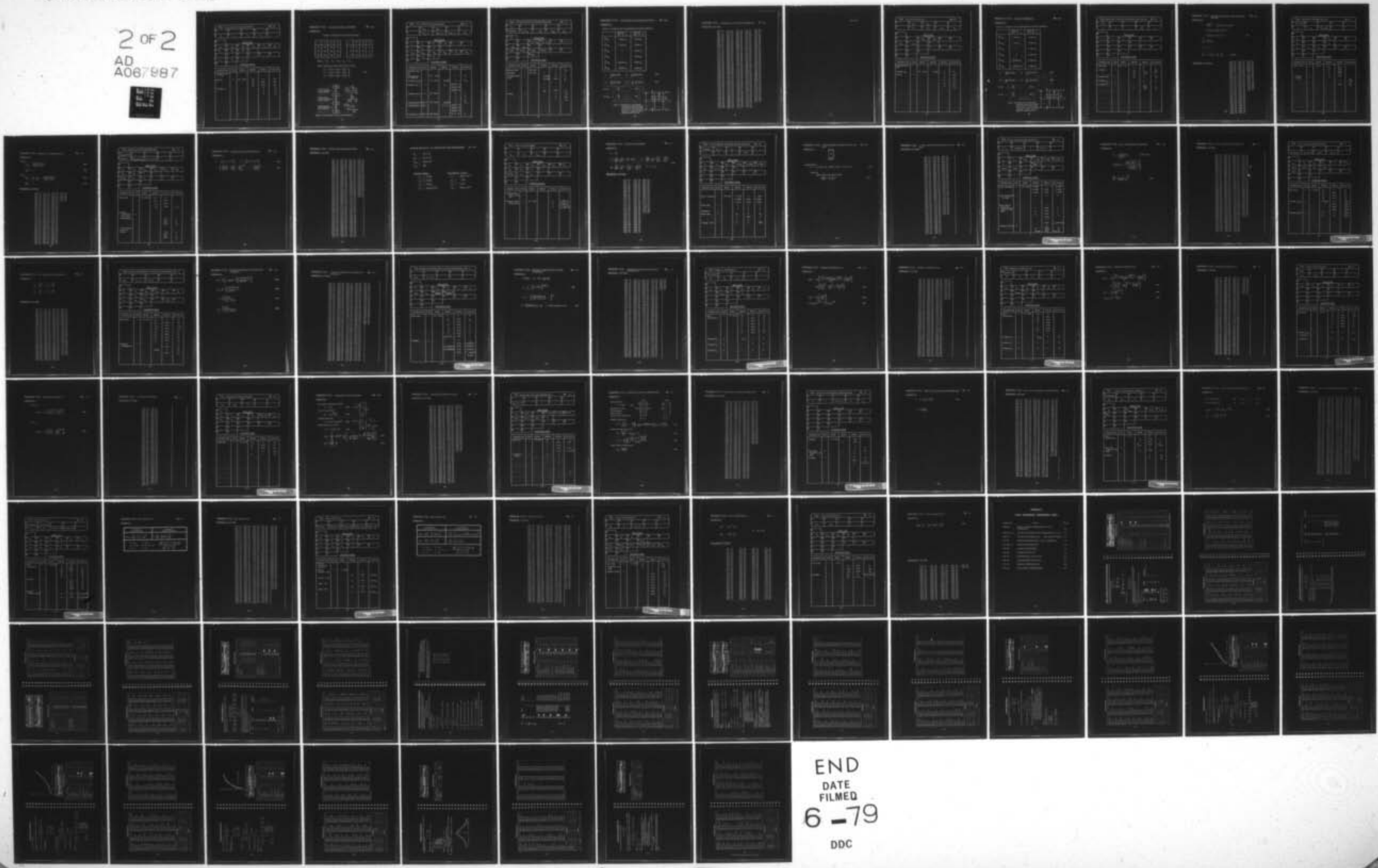
FORMULAS:

TABLE 33 FORMULAS FOR FLEXURAL RIGIDITY WITH ISOTROPIC SUBSTRUCTURE

| | $\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_1$ | $\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_2$ | $R_{1D}$ | $R_{2D}$ |
|---|---|---|---|---|
| $\frac{12}{h^3}D'_{11}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$ | 1 | 1 | $\cos 2(\theta-\delta_1)$ | $\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{22}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$ | 1 | 1 | $-\cos 2(\theta-\delta_1)$ | $\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{12}-\left(\frac{2z_o}{h}\right)^3 Q^o_{12}$ | 1 | -1 | 0 | $-\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{66}-\frac{1}{2}\left(\frac{2z_o}{h}\right)^3 (Q^o_{11}-Q^o_{12})$ | 0 | 1 | 0 | $-\cos 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{16}$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $-\sin 4(\theta-\delta_2)$ |
| $\frac{12}{h^3}D'_{26}$ | 0 | 0 | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $\sin 4(\theta-\delta_2)$ |

AD-A067 987 AIR FORCE MATERIALS LAB WRIGHT-PATTERSON AFB OH F/G 9/2
SR-52 MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS--ETC(U)
JAN 79 S W TSAI, H T HAHN, F HUBER
UNCLASSIFIED AFML-TR-77-50 NL

2 OF 2
AD
A067987



END
DATE
FILMED
6-79
DDC

TITLE: CURVATURE DUE TO BENDING NO: 090

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $M_i$ | B $k_1$ | C $k_2$ | D $k_6$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $d_{11}$ | 02 $d_{22}$ | 03 $d_{12}$ | 04 $d_{66}$ | 05 $d_{16}$ | 06 $d_{26}$ |
| 07 $M_1$ | 08 $M_2$ | 09 $M_6$ | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 $k_1$ | 18 $k_2$ | 19 $k_6$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from VI-1b | | | | | |
| Input $M_i$ | 6 | 019 A&B | $M_1$ | A | $M_1$ |
| | | | $M_2$ | RUN | $M_2$ |
| | | | $M_6$ | RUN | $M_6$ |
| Compute $k_i$ | | | | B | $k_1$ |
| | | | | C | $k_2$ |
| | | | | D | $k_6$ |

## PROGRAM TITLE: CURVATURE DUE TO BENDING    NO: 090

## FORMULAS:

TABLE 50 FLEXURE-CURVATURE RELATIONS

| | $k_1$ | $k_2$ | $k_6$ |
|---|---|---|---|
| $M_1$ | $D_{11}$ | $D_{12}$ | $D_{16}$ |
| $M_2$ | $D_{21}$ | $D_{22}$ | $D_{26}$ |
| $M_6$ | $D_{61}$ | $D_{62}$ | $D_{66}$ |

| | $M_1$ | $M_2$ | $M_6$ |
|---|---|---|---|
| $k_1$ | $d_{11}$ | $d_{12}$ | $d_{16}$ |
| $k_2$ | $d_{21}$ | $d_{22}$ | $d_{26}$ |
| $k_6$ | $d_{61}$ | $d_{62}$ | $d_{66}$ |

Since $e_1 = zk_1$, $e_2 = zk_2$, $e_6 = zk_6$

Finally, from stress-strain relation of the t-th layer.

$$\left.\begin{aligned} \sigma_1 &= Q_{11}e_1 + Q_{12}e_2 + Q_{16}e_6 \\ \sigma_2 &= Q_{21}e_1 + Q_{22}e_2 + Q_{26}e_6 \\ \sigma_6 &= Q_{61}e_1 + Q_{62}e_2 + Q_{66}e_6 \end{aligned}\right\} \qquad (281)$$

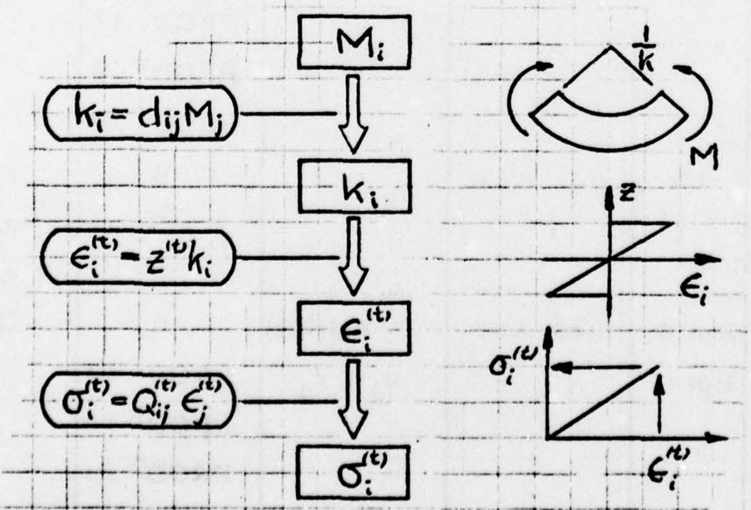


Figure 73 Flow diagram for ply stress calculation.

TITLE: PLY STRESS DUE TO BENDING NO: 092

| | | | | |
|---|---|---|---|---|
| A' | B' $Q_{16}$ | C' $Q_{26}$ | D' | E' |
| A | B $Q_{11}, \sigma_1$ | C $Q_{22}, \sigma_2$ | D $Q_{12}, \sigma_6$ | E $Q_{66}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $I_1$ | 02 $I_2$ | 03 $R_1$ | 04 $R_2$ | 05 | 06 |
| 07 $\sigma_1$ | 08 $\sigma_2$ | 09 $\sigma_6$ | 10 | | |
| 11 $Q_{11}$ | 12 $Q_{22}$ | 13 $Q_{12}$ | 14 $Q_{66}$ | 15 $Q_{16}$ | 16 $Q_{26}$ |
| 17 $\epsilon_1$ | 18 $\epsilon_2$ | 19 $\epsilon_6$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from 090 | | | | | |
| Input ply data & initialize | 7 | 045 A&B | | A | $E_L$ |
| Compute $Q_{ij}(\alpha_1)$ | 8 | 041 A&B | $\alpha_1$(deg) | B | $Q_{11}$ |
| Compute $\epsilon_i(z_1)$ | 9 | | $z_1$ | PROD* 17 | |
| | | | | PROD* 18 | |
| | | | | PROD* 19 | |
| Compute $\sigma_i(z_1)$ | 10 | 023 A&B | | B | $\sigma_1$ |
| | | | | C | $\sigma_2$ |
| | | | | D | $\sigma_6$ |
| Compute $Q_{ij}(\alpha_2)$ | Rpt 8 | 023 A&B | $\alpha_2$(deg) | B | $Q_{11}$ |
| Compute $\epsilon_i(z_2)$ | Rpt 9 | | $z_2$ / $z_1$ | PROD* 17 | |
| | | | | PROD* 18 | |
| | | | | PROD* 19 | |
| Compute $\sigma_i(z_2)$ | Rpt 10 | 023 A&B | | B, C, D | $\sigma_i$ |

TITLE: INVARIANTS OF COUPLING MODULUS NO: 093

| | | | | |
|---|---|---|---|---|
| A' $\sigma_i$ | B' | C' | D' | E' |
| A Initialize | B $a_t$, $R_{1B}$ | C $\delta_{1B}$ | D $R_{2B}$ | E $\delta_{2B}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $-2a_t$ | | | | | |
| 01 | 02 | 03 $R_1$, $R_{1B}$ | 04 $R_2$, $R_{2B}$ | 05 t | 06 2t - 1 |
| 07 $\delta_{1B}$ | 08 $\delta_{2B}$ | 09 | 10 | | |
| 11 Fc2B | 12 Fs2B | 13 Fc4B | 14 Fs4B | 15 | 16 |
| 17 | 18 | 19 $-4a_t$ | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input ply data | 1 | 045 A&B | | A | $E_L$ |
| Input data | 2 | 093 A&B | n | A | n |
| Compute invariants | 3 | | $a_1$(deg) | B | $a_1$ |
| | | | $a_2$(deg) | RUN | $a_2$ |
| | | | . | . | . |
| | | | . | . | . |
| | | | . | . | . |
| | | | $a_N$ | RUN | $R_{1B}$ |
| Recall | 4 | | | C | $\delta_{1B}$ |
| | | | | D | $R_{2B}$ |
| | | | | E | $\delta_{2B}$ |

**PROGRAM TITLE:** INVARIANTS OF COUPLING MODULUS **NO:** 093

**FORMULAS:**

TABLE 56 FORMULA FOR TRANSFORMED COUPLING MODULUS

| | $\sqrt{V_1^2+V_3^2}\quad R_1$ | $\sqrt{V_2^2+V_4^2}\quad R_2$ |
|---|---|---|
| $\frac{h^2}{2}B'_{11}$ | $\cos 2(\theta-\delta_1)$ | $\cos 4(\theta-\delta_2)$ |
| $\frac{h^2}{2}B'_{22}$ | $-\cos 2(\theta-\delta_1)$ | $\cos 4(\theta-\delta_2)$ |
| $\frac{h^2}{2}B'_{12}$ | 0 | $-\cos 4(\theta-\delta_2)$ |
| $\frac{h^2}{2}B'_{66}$ | 0 | $-\cos 4(\theta-\delta_2)$ |
| $\frac{h^2}{2}B'_{16}$ | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $-\sin 4(\theta-\delta_2)$ |
| $\frac{h^2}{2}B'_{26}$ | $-\frac{1}{2}\sin 2(\theta-\delta_1)$ | $\sin 4(\theta-\delta_2)$ |

$$V_1 = \frac{1}{N^2}\sum F_t \cos 2\alpha, \qquad V_2 = \frac{1}{N^2}\sum F_t \cos 4\alpha \qquad (322)$$

$$V_3 = -\frac{1}{N^2}\sum F_t \sin 2\alpha \qquad V_4 = -\frac{1}{N^2}\sum F_t \sin 4\alpha \qquad (323)$$

$$\tan 2\delta_1 = -\frac{V_3}{V_1} \qquad (320)$$

$$\tan 4\delta_2 = = -\frac{V_4}{V_2} \qquad (321)$$

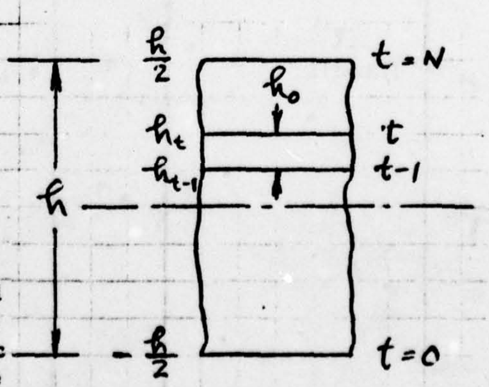


Figure 78 Integration of unsymmetric laminates must be performed for the entire thickness from -h/2 to h/2. (For symmetric laminates, integration can be limited to from 0 to h/2).

**PROGRAM TITLE:** INVARIANTS OF COUPLING MODULUS **NO:** 093

**PROGRAM LISTING:**

```
000 46   045 02   090 65   135 43   180 85
001 11   046 95   091 43   136 01   181 43
002 42   047 42   092 00   137 04   182 01
003 09   048 01   093 06   138 55   183 02
004 09   049 09   094 95   139 43   184 40
005 00   050 43   095 44   140 01   185 95
006 42   051 00   096 01   141 03   186 30
007 01   052 00   097 04   142 94   187 55
008 01   053 33   098 43   143 95   188 53
009 42   054 65   099 09   144 22   189 43
010 01   055 43   100 09   145 34   190 09
011 02   056 00   101 75   146 55   191 09
012 42   057 06   102 43   147 04   192 65
013 01   058 95   103 00   148 95   193 02
014 03   059 44   104 05   149 42   194 54
015 42   060 01   105 95   150 00   195 40
016 01   061 01   106 90   151 08   196 95
017 04   062 43   107 87   152 43   197 49
018 42   063 01   108 43   153 01   198 00
019 00   064 09   109 00   154 03   199 03
020 05   065 33   110 05   155 40   200 43
021 46   066 65   111 85   156 85   201 00
022 88   067 43   112 01   157 43   202 03
023 42   068 00   113 95   158 01   203 81
024 00   069 06   114 41   159 04   204 46
025 05   070 95   115 88   160 40   205 13
026 65   071 44   116 46   161 95   206 43
027 02   072 01   117 87   162 30   207 00
028 75   073 03   118 43   163 55   208 07
029 01   074 43   119 01   164 53   209 81
030 95   075 00   120 02   165 43   210 46
031 42   076 00   121 55   166 09   211 14
032 00   077 32   122 43   167 09   212 43
033 06   078 65   123 01   168 65   213 00
034 81   079 43   124 01   169 02   214 04
035 46   080 00   125 94   170 54   215 81
036 12   081 06   126 95   171 40   216 46
037 65   082 95   127 22   172 95   217 15
038 02   083 44   128 34   173 49   218 43
039 94   084 01   129 55   174 00   219 00
040 95   085 02   130 02   175 04   220 08
041 42   086 43   131 95   176 43   221 81
042 00   087 01   132 42   177 01
043 00   088 09   133 00   178 01
044 65   089 32   134 07   179 40
```

NO: 093

TITLE: COUPLING MODULUS NO: 097

| | | | | |
|---|---|---|---|---|
| A' | B' $B_{16}$ | C' $B_{26}$ | D' | E' |
| A | B $B_{11}$ | C $B_{22}$ | D $B_{12}$ | E $B_{66}$ |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 | 02 | 03 $R_{1B}$ | 04 $R_{2B}$ | 05 | 06 |
| 07 $\delta_{1B}$ | 08 $\delta_{2B}$ | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Continue from 093 | | | | | |
| Compute $B_{ij}$ | 3 | 041 A&B | θ (deg) | B | $B_{11}$ |
| Recall | 3' | | | C | $B_{22}$ |
| | | | | D | $B_{12}$ |
| | | | | E | $B_{66}$ |
| | | | | B' | $B_{16}$ |
| | | | | C' | $B_{26}$ |

PROGRAM TITLE: COUPLING MODULUS NO: 097

FORMULAS:

TABLE 56 FORMULA FOR TRANSFORMED COUPLING MODULUS

|  | $\sqrt{V_1^2 + V_3^2} \quad R_1$ | $\sqrt{V_2^2 + V_4^2} \quad R_2$ |
|---|---|---|
| $\frac{h^2}{2} B'_{11}$ | $\cos 2(\theta - \delta_1)$ | $\cos 4(\theta - \delta_2)$ |
| $\frac{h^2}{2} B'_{22}$ | $-\cos 2(\theta - \delta_1)$ | $\cos 4(\theta - \delta_2)$ |
| $\frac{h^2}{2} B'_{12}$ | 0 | $-\cos 4(\theta - \delta_2)$ |
| $\frac{h^2}{2} B'_{66}$ | 0 | $-\cos 4(\theta - \delta_2)$ |
| $\frac{h^2}{2} B'_{16}$ | $-\frac{1}{2}\sin 2(\theta - \delta_1)$ | $-\sin 4(\theta - \delta_2)$ |
| $\frac{h^2}{2} B'_{26}$ | $-\frac{1}{2}\sin 2(\theta - \delta_1)$ | $\sin 4(\theta - \delta_2)$ |

$$V_1 = \frac{1}{N^2}\sum F_t \cos 2\alpha, \qquad V_2 = \frac{1}{N^2}\sum F_t \cos 4\alpha \qquad (322)$$

$$V_3 = -\frac{1}{N^2}\sum F_t \sin 2\alpha \qquad V_4 = -\frac{1}{N^2}\sum F_t \sin 4\alpha \qquad (323)$$

$$\tan 2\delta_1 = -\frac{V_3}{V_1} \qquad (320)$$

$$\tan 4\delta_2 = = -\frac{V_4}{V_2} \qquad (321)$$

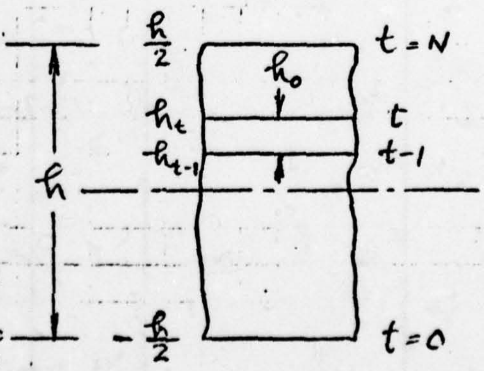


Figure 78 Integration of unsymmetric laminates must be performed for the entire thickness from -h/2 to h/2. (For symmetric laminates, integration can be limited to from 0 to h/2).

TITLE: MOISTURE CONTENT AND SWELLING STRAIN NO: 099

| | | | | |
|---|---|---|---|---|
| A' $e_T^H$ | B' | C' | D' | E' |
| A s | B $v_v$ | C c | D $\alpha_T^H$ | E $c_o$ |

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 01 s | 02 $v_v$ | 03 C | 04 | 05 | 06 |
| 07 | 08 | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input data | 1 | 099 A | s | A | s |
| | | | $v_v$ | B | $v_v$ |
| Compute c | 2 | | $c_m$ | C | $c_m$ |
| | | | $v_m$ | RUN | $c_m c_v$ |
| | | | $s_m$ | RUN | c |
| Compute $\alpha_T^H$ | 3 | | $\nu_m$ | D | $\alpha_T^H$ |
| Compute $c_o$ | 4 | | | E | $c_o$ |
| Compute $e_T^H$ | 5 | | | A' | |
| | | | $\rho$ | RUN | |
| | | | $\rho'$ | RUN | $e_T^H$ |

PROGRAM TITLE: MOISTURE CONTENT AND SWELLING STRAIN NO: 099

FORMULAS:

$$c = \frac{M' - M}{M} = c_m m_m + c_f m_f + M_{vw} / M$$

$$= (c_m v_m \rho_m + c_f v_f \rho_f + v_v \rho_w) / \rho$$

$$= (c_m v_m s_m + c_f v_f s_f + v_v) / s \qquad (374)$$

$$\alpha_T^H = \frac{1 + \nu_m}{3} s$$

$$c_o = v_v / s$$

$$e_T^H = \frac{1}{2} \left[ (1 + c) \frac{\rho}{\rho'} - 1 \right] \qquad \text{cf. (402)}$$

PROGRAM LISTING:

```
000 46   030 00   060 01
001 11   031 03   061 85
002 42   032 81   062 43
003 00   033 46   063 00
004 01   034 14   064 03
005 81   035 85   065 54
006 46   036 01   066 65
007 12   037 95   067 81
008 42   038 55   068 55
009 00   039 03   069 81
010 02   040 65   070 75
011 81   041 43   071 01
012 46   042 00   072 95
013 13   043 01   073 55
014 81   044 95   074 02
015 65   045 81   075 95
016 81   046 46   076 81
017 65   047 15
018 81   048 43
019 85   049 00
020 43   050 02
021 00   051 55
022 02   052 43
023 95   053 00
024 55   054 01
025 43   055 95
026 00   056 81
027 01   057 46
028 95   058 16
029 42   059 53
```

| TITLE: RESIDUAL STRESSES (MICRO) | | | | NO: 101 |
|---|---|---|---|---|
| A' Input Data | B' | C' | D' | E' |
| A $e_f$ | B $e_m$ | C $\bar{\sigma}^R_{mL}, \bar{\sigma}^R_{fL}$ | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_f$ | 02 | 03 | 04 $E_m$ | 05 | 06 |
| 07 $\eta_1$ | 08 | 09 | 10 $v_f$ | | |
| 11 $v_m$ | 12 $e_f$ | 13 $e_m$ | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 101 A | $E_f$ | A' | |
| | | | $E_m$ | RUN | |
| | | | $\eta_1$ | RUN | |
| | | | $v_f$ | RUN | |
| | | | $e_f$ | RUN | |
| Compute stress | 2 | | $e_m$ | RUN | |
| | | | | A | $\bar{\sigma}^R_{mL}$ |
| | | | | RUN | $\bar{\sigma}^R_{fL}$ |

## PROGRAM TITLE: RESIDUAL STRESSES (MICRO) NO: 101

## FORMULAS:

**Matrix**

$$\bar{\sigma}^{R}_{mL} = \frac{v_f E_f E_m (e_f - \eta_1 e_m)}{\eta_1 v_f E_f + v_m E_m} \tag{387}$$

$$\bar{\sigma}^{R}_{mT} = 0 \tag{388}$$

**Fiber**

$$\bar{\sigma}^{R}_{fL} = -\frac{v_m}{v_f}\,\bar{\sigma}^{R}_{mL} = \frac{v_m E_m E_f (\eta_1 e_m - e_f)}{\eta_1 v_f E_f + v_m E_m} \tag{389}$$

$$\bar{\sigma}^{R}_{fT} = 0 \tag{390}$$

## PROGRAM LISTING:

```
000 46   030 01   060 55   090 01
001 16   031 03   061 53   091 00
002 42   032 81   062 43   092 94
003 00   033 46   063 00   093 95
004 01   034 11   064 07   094 81
005 81   035 43   065 65
006 42   036 01   066 43
007 00   037 00   067 01
008 04   038 65   068 00
009 81   039 43   069 65
010 42   040 00   070 43
011 00   041 01   071 00
012 07   042 65   072 01
013 81   043 43   073 85
014 42   044 00   074 43
015 01   045 04   075 01
016 00   046 65   076 01
017 94   047 53   077 65
018 85   048 43   078 43
019 01   049 01   079 00
020 95   050 02   080 04
021 42   051 75   081 54
022 01   052 43   082 95
023 01   053 00   083 81
024 81   054 07   084 65
025 42   055 65   085 43
026 01   056 43   086 01
027 02   057 01   087 01
028 81   058 03   088 55
029 42   059 54   089 43
```

TITLE: STRESS ANALYSIS PARAMETERS NO: 103

| | | | | |
|---|---|---|---|---|
| A' Eng'g Const. | B' | C' | D' | E' |
| A Parallel | B Perpendicular | C k, n, et al | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_L$ | 02 $E_T$ | 03 $G_{LT}$ | 04 $\nu_{LT}$ | 05 $S_{11}$ | 06 $S_{22}$ |
| 07 $S_{66}$ | 08 $S_{12}$ | 09 k | 10 n | | |
| 11 $\sqrt{n^2-4k}$ | 12 $\mu_1$ | 13 $\mu_2$ | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 103 A&B | $E_L$ | A' | |
| | | | $E_T$ | RUN | |
| | | | $G_{LT}$ | RUN | |
| | | | $\nu_{LT}$ | RUN | $S_{12}$ |
| Compute parameters | 2 | | | | |
| Crack parallel to fibers | | | | A | k |
| | | | | RUN | n |
| | | | | RUN | $\mu_1$ |
| | | | | RUN | $\mu_2$ |
| Crack perpendicular to fibers | | | | B | k |
| | | | | RUN | n |
| | | | | RUN | $\mu_1$ |
| | | | | RUN | $\mu_2$ |

**PROGRAM TITLE:** STRESS ANALYSIS PARAMETERS **NO:** 103

**FORMULAS:**

$$\mu_1 = \frac{1}{2}\left[n + (n^2 - 4k)^{1/2}\right] , \quad \mu_2 = \frac{1}{2}\left[n - (n^2 - 4k)^{1/2}\right] \quad (511)$$

$$n = \left[2\left(\sqrt{\frac{S_{22}}{S_{11}}} + \frac{S_{12}}{S_{11}}\right) + \frac{S_{66}}{S_{11}}\right]^{1/2} , \quad k = \left(\frac{S_{22}}{S_{11}}\right)^{1/2} \quad (512)$$

**PROGRAM TITLE:** STRESS ANALYSIS PARAMETERS **NO:** 103

**PROGRAM LISTING:**

```
000 46   045 95   090 81   135 01
001 16   046 94   091 40   136 20
002 42   047 42   092 75   137 42
003 00   048 00   093 04   138 00
004 01   049 08   094 65   139 06
005 81   050 81   095 43   140 43
006 42   051 46   096 00   141 00
007 00   052 11   097 09   142 02
008 02   053 43   098 95   143 20
009 81   054 00   099 30   144 42
010 42   055 06   100 42   145 00
011 00   056 55   101 01   146 05
012 03   057 43   102 01   147 43
013 81   058 00   103 85   148 00
014 42   059 05   104 43   149 03
015 00   060 95   105 01   150 20
016 04   061 30   106 00   151 42
017 43   062 42   107 95   152 00
018 00   063 00   108 95   153 07
019 01   064 09   109 55   154 41
020 20   065 81   110 02   155 11
021 42   066 85   111 95   156 81
022 00   067 43   112 42
023 05   068 00   113 01
024 43   069 08   114 02
025 00   070 55   115 81
026 02   071 43   116 43
027 20   072 00   117 01
028 42   073 05   118 00
029 00   074 95   119 75
030 06   075 65   120 43
031 43   076 02   121 01
032 00   077 85   122 01
033 03   078 43   123 95
034 20   079 00   124 55
035 42   080 07   125 02
036 00   081 55   126 95
037 07   082 43   127 42
038 43   083 00   128 01
039 00   084 05   129 03
040 04   085 95   130 81
041 55   086 30   131 46
042 43   087 42   132 12
043 00   088 01   133 43
044 01   089 00   134 00
```

SAMPLE PROBLEM: 103 STRESS ANALYSIS PARAMETERS NO: 103

$E_L = 181 \times 10^9$

$E_T = 10.3 \times 10^9$

$G_{LT} = 7.17 \times 10^9$

$\nu_{LT} = .28$

**Parallel to fibers**

$k = 4.192$

$n = 5.750$

$\mu_1 = 4.894$

$\mu_2 = 856.6 \times 10^{-3}$

**Perpendicular to fibers**

$k = 238.5 \times 10^{-3}$

$n = 1.372$

$\mu_1 = 1.167$

$\mu_2 = 204.3 \times 10^{-3}$

TITLE: CRACK TIP STRESSES NO: 107

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $\sigma_x$ | B $\sigma_y$ | C $\tau_{xy}$ | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_L$ | 02 $E_T$ | 03 $G_{LT}$ | 04 $\nu_{LT}$ | 05 $S_{11}$ | 06 $S_{12}$ |
| 07 $S_{66}$ | 08 $S_{12}$ | 09 k | 10 n | | |
| 11 $\sqrt{n^2-4k}$ | 12 $\mu_1$ | 13 $\mu_2$ | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Compute parameters with 103 | 1 | | | | |
| Compute crack tip stresses | 2 | 107 A&B | | A<br>B<br>C | $\sigma_x\sqrt{2\pi r}/K_I$<br>$\sigma_y\sqrt{2\pi r}/K_I$<br>$\tau_{xy}\sqrt{2\pi r}/K_I$ |

PROGRAM TITLE: CRACK TIP STRESSES NO: 107

FORMULAS:

$$\theta = 90^{o}$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\,\frac{k}{\sqrt{2}\,\bar{\mu}}\left(\sqrt{\mu_1} - \sqrt{\mu_2}\right), \qquad \sigma_y = \frac{K_I}{\sqrt{2\pi r}}\,\frac{1}{\sqrt{2}\,\bar{\mu}}\left(\frac{\mu_1}{\sqrt{\mu_2}} - \frac{\mu_2}{\sqrt{\mu_1}}\right) \qquad (518)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\,\frac{k}{\sqrt{2}\,\bar{\mu}}\left(\frac{1}{\sqrt{\mu_1}} - \frac{1}{\sqrt{\mu_2}}\right), \qquad \bar{\mu} = \mu_1 - \mu_2$$

PROGRAM LISTING:

```
000 46   030 75   060 54
001 11   031 43   061 95
002 43   032 01   062 46
003 00   033 03   063 87
004 09   034 55   064 55
005 65   035 43   065 02
006 53   036 01   066 30
007 43   037 02   067 55
008 01   038 30   068 53
009 02   039 51   069 43
010 30   040 87   070 01
011 75   041 81   071 02
012 43   042 46   072 75
013 01   043 13   073 43
014 03   044 43   074 01
015 30   045 00   075 03
016 54   046 09   076 54
017 51   047 65   077 95
018 87   048 53   078 56
019 81   049 43
020 46   050 01
021 12   051 02
022 43   052 30
023 01   053 20
024 02   054 75
025 55   055 43
026 43   056 01
027 01   057 03
028 03   058 30
029 30   059 20
```

TITLE: Y FOR CENTER-CRACKED PLATE AND COD NO: 109

| | | | | |
|---|---|---|---|---|
| A' Constants | B' $E_L$, $\gamma$ | C' $\tau_y$ | D' | E' |
| A a | B w | C Y | D COD | E |

## REGISTER

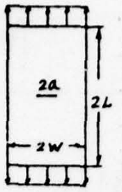
| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 a | 02 w | 03 a/w | 04 1.5254 | 05 -0.2881 | 06 0.1282 |
| 07 $E_L$ | 08 $\gamma = 2n$ | 09 $\tau_y$ | 10 Y | | |
| 11 $\sigma$ | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter constants | 1 | 109 A&B | 0.1282 | A' | .1282 |
| | | | -0.2881 | RUN | -0.2881 |
| | | | 1.5254 | RUN | 1.5254 |
| Enter data | 2 | | a | A | a |
| | | | w | B | w |
| Compute Y | 3 | | | C | Y |
| Enter data | 4 | | $E_L$ | B' | $E_L$ |
| | | | $\gamma$ | RUN | $\gamma$ |
| | | | $\tau_y$ | C' | $\tau_y$ |
| Compute COD | 5 | | $\sigma$ | D | COD |

**PROGRAM TITLE:** Y FOR CENTER-CRACKED PLATE AND COD **NO:** 109

**FORMULAS:**



**Isotropic plates**

$$Y = 1 + 0.1282\,(a/w) - 0.2881\,(a/w)^2 + 1.5254\,(a/w)^3 \qquad (516)$$

**Total COD**

COD = elastic COD + plastic CTOD

$$= \frac{Y a \sigma}{E_L}\left(\gamma + \frac{\pi}{2}\,\frac{Y}{\tau_y}\,\sigma\right) \qquad (526)$$

**PROGRAM TITLE:** Y FOR CENTER-CRACKED PLATE AND COD **NO:** 109

**PROGRAM LISTING:**

```
000 46   045 06   090 09
001 11   046 85   091 54
002 42   047 01   092 95
003 00   048 95   093 81
004 01   049 42   094 46
005 81   050 01   095 16
006 46   051 00   096 42
007 12   052 81   097 00
008 42   053 46   098 06
009 00   054 14   099 81
010 02   055 42   100 42
011 81   056 01   101 00
012 46   057 01   102 05
013 13   058 65   103 81
014 55   059 43   104 42
015 43   060 01   105 00
016 00   061 00   106 04
017 01   062 65   107 81
018 95   063 43   108 46
019 20   064 00   109 17
020 42   065 01   110 42
021 00   066 55   111 00
022 03   067 43   112 07
023 45   068 00   113 81
024 03   069 07   114 42
025 65   070 65   115 00
026 43   071 53   116 08
027 00   072 43   117 81
028 04   073 00   118 46
029 85   074 08   119 18
030 43   075 85   120 42
031 00   076 59   121 00
032 03   077 55   122 09
033 40   078 02   123 81
034 65   079 65
035 43   080 43
036 00   081 01
037 05   082 00
038 85   083 65
039 43   084 43
040 00   085 01
041 03   086 01
042 65   087 55
043 43   088 43
044 00   089 00
```

TITLE: ECDZ AND NOTCHED STRENGTH NO: 113

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A Y, $c_o$ | B Constants | C X | D $YX_n/X$ | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 a | 02 w | 03 a/w | 04 1.5254 | 05 -0.2881 | 06 0.1282 |
| 07 X | 08 | 09 | 10 $(\frac{x}{YX_n})^2-1$ | | |
| 11 used | 12 used | 13 $c_o$ | 14 a | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter constants | 1 | 113 A&B | 0.1282 | A | 0.1282 |
| | | | -0.2881 | RUN | -0.2881 |
| | | | 1.5254 | RUN | 1.5254 |
| Enter unnotched strength | 2 | | X | B | X |
| Enter data to determine $c_o$ Repeat for all data | 3 | | | C | |
| | | | a | RUN | a |
| | | | w | RUN | Y |
| | | | $X_n$ | RUN | $c_o$ |
| | | | | st flg | |
| | | | | 6 | |
| | | | | RUN | $c_o$ (best fit) |
| Compute $YX_n/X$ | 4 | | | D | |
| | | | $c_o$ | RUN | $c_o$ |
| | | | a | RUN | a |
| | | | $\theta$(deg) | RUN | $YX_n/X$ |

PROGRAM TITLE: ECDZ AND NOTCHED STRENGTH NO: 113

FORMULAS:

$$c_o = \frac{a}{\left(\frac{X}{YX_n}\right)^2 - 1}$$

cf. Eq. (540)

$$c_o\,(\text{best fit}) = \frac{\sum a\left[\left(\frac{X}{YX_n}\right)^2 - 1\right]}{\sum\left[\left(\frac{X}{YX_n}\right)^2 - 1\right]^2}$$

$$\frac{YX_n}{X} = \left(\frac{c_o}{a\sin\theta + c_o}\right)^{1/2} \qquad (546)$$

**PROGRAM TITLE:** ECDZ AND NOTCHED STRENGTH **NO:** 113

**PROGRAM LISTING:**

```
000 46   045 00   090 01   135 01
001 13   046 03   091 00   136 02
002 22   047 65   092 95   137 81
003 50   048 43   093 81   138 50
004 00   049 00   094 60   139 00
005 42   050 06   095 00   140 81
006 01   051 85   096 01   141 46
007 01   052 01   097 00   142 14
008 42   053 95   098 06   143 81
009 01   054 65   099 41   144 42
010 02   055 81   100 00   145 01
011 81   056 95   101 01   146 03
012 42   057 20   102 02   147 81
013 00   058 65   103 43   148 42
014 01   059 43   104 01   149 01
015 81   060 00   105 02   150 04
016 42   061 07   106 55   151 81
017 00   062 95   107 43   152 32
018 02   063 40   108 01   153 65
019 55   064 75   109 01   154 43
020 43   065 01   110 95   155 01
021 00   066 95   111 81   156 04
022 01   067 42   112 46   157 85
023 95   068 01   113 11   158 43
024 20   069 00   114 42   159 01
025 42   070 40   115 00   160 03
026 00   071 44   116 06   161 95
027 03   072 01   117 81   162 20
028 45   073 01   118 42   163 65
029 03   074 43   119 00   164 43
030 65   075 00   120 05   165 01
031 43   076 01   121 81   166 03
032 00   077 65   122 42   167 95
033 04   078 43   123 00   168 30
034 85   079 01   124 04   169 81
035 43   080 00   125 81   170 00
036 00   081 95   126 46
037 03   082 44   127 12
038 40   083 01   128 42
039 65   084 02   129 00
040 43   085 43   130 07
041 00   086 00   131 81
042 05   087 01   132 01
043 85   088 55   133 01
044 43   089 43   134 42
```

TITLE: OFF-AXIS FATIGUE STRENGTH NO: 117

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A Constants | B $a_\theta$, $b_\theta$, etal | C $S_\theta$ (S) | D $S_\theta$ (T) | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $a_S$ | 02 $b_S$ | 03 $a_T$ | 04 $b_T$ | 05 $\theta$ | 06 1/mn |
| 07 $a_\theta(S)$ | 08 $b_\theta(S)$ | 09 $m^2$ | 10 $a_\theta(T)$ | | |
| 11 $b_\theta(T)$ | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 117 A&B | $a_S$ | A | $a_S$ |
| | | | $b_S$ | RUN | $b_S$ |
| | | | $a_T$ | RUN | $a_T$ |
| | | | $b_T$ | RUN | $b_T$ |
| Compute $a_\theta$, $b_\theta$ | 2 | | $\theta$ (deg) | B | $a_\theta(S)$ |
| | | | | RUN | $b_\theta(S)$ |
| | | | | RUN | $a_\theta(T)$ |
| | | | | RUN | $b_\theta(T)$ |
| Compute $S_\theta$ at N | 3 | | N | C | $S_\theta(S)$ |
| | | | N | D | $S_\theta(T)$ |

PROGRAM TITLE: OFF-AXIS FATIGUE STRENGTH NO: 117

FORMULAS:

$$a_\theta = \frac{a_s}{mn} \quad , \quad b_\theta = \frac{b_s}{mn}$$

$$a_\theta = \frac{a_s}{n^2} \quad , \quad b_\theta = \frac{b_s}{n^2}$$

PROGRAM LISTING:

```
000 46   030 20   060 42   090 65
001 11   031 42   061 00   091 43
002 42   032 00   062 09   092 00
003 00   033 06   063 43   093 07
004 01   034 65   064 00   094 95
005 81   035 43   065 03   095 94
006 42   036 00   066 55   096 85
007 00   037 01   067 43   097 43
008 02   038 95   068 00   098 00
009 81   039 42   069 09   099 08
010 42   040 00   070 95   100 95
011 00   041 07   071 42   101 81
012 03   042 81   072 01   102 46
013 81   043 43   073 00   103 14
014 42   044 00   074 81   104 28
015 00   045 06   075 43   105 65
016 04   046 65   076 00   106 43
017 81   047 43   077 04   107 01
018 46   048 00   078 55   108 00
019 12   049 02   079 43   109 95
020 42   050 95   080 00   110 94
021 00   051 42   081 09   111 85
022 05   052 00   082 95   112 43
023 33   053 08   083 42   113 01
024 65   054 81   084 01   114 01
025 43   055 43   085 01   115 95
026 00   056 00   086 81   116 81
027 05   057 05   087 46
028 32   058 32   088 13
029 95   059 40   089 28
```

TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE NO: 119

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A Data | B $u_1, u_2$ | C $k_S, k_T$ | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $E_L$ | 02 $E_T$ | 03 $G_{LT}$ | 04 $\nu_{LT}$ | 05 $a_S$ | 06 $b_S$ |
| 07 $a_T$ | 08 $b_T$ | 09 $2\theta$ | 10 $u_1$ | | |
| 11 $u_2$ | 12 $k_S$ | 13 $k_T$ | 14 | 15 used | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 119 A&B | $E_L$ | A | $E_L$ |
| | | | $E_T$ | RUN | $E_T$ |
| | | | $G_{LT}$ | RUN | $G_{LT}$ |
| | | | $\nu_{LT}$ | RUN | $\nu_{LT}$ |
| | | | $a_S$ | RUN | $a_S$ |
| | | | $b_S$ | RUN | $b_S$ |
| | | | $a_T$ | RUN | $a_T$ |
| | | | $b_T$ | RUN | $b_T$ |
| Compute parameters | 2 | | | B | $u_1$ |
| | | | | RUN | $u_2$ |
| | | | $\theta$(deg) | C | $k_S$ |
| | | | | RUN | $k_T$ |

PROGRAM TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE NO: 119

FORMULAS:

$$k_T = \frac{1}{2}\left[1 - \sec 2\theta + \frac{(u_1 + \sec 2\theta)\tan^2 2\theta}{u_2 + \tan^2 2\theta}\right] \quad (555)$$

$$k_S = -\frac{1}{2}\frac{(u_1 + \sec 2\theta)\tan 2\theta}{u_2 + \tan^2 2\theta} \quad (556)$$

$$u_1 = \frac{1 - E_L/E_T}{1 + 2\nu_{LT} + E_L/E_T} \quad (557)$$

$$u_2 = \frac{E_L/G_{LT}}{1 + 2\nu_{LT} + E_L/E_T} \quad (558)$$

**PROGRAM TITLE:** FATIGUE STRENGTH OF ANGLE-PLY LAMINATE **NO:** 119

**PROGRAM LISTING:**

```
000 46   045 95   090 13   135 95   180 09
001 11   046 55   091 65   136 94   181 32
002 42   047 53   092 02   137 55   182 40
003 00   048 01   093 95   138 02   183 55
004 01   049 85   094 42   139 95   184 43
005 81   050 02   095 00   140 42   185 01
006 42   051 65   096 09   141 01   186 06
007 00   052 43   097 53   142 03   187 75
008 02   053 00   098 43   143 81   188 01
009 81   054 04   099 01   144 01   189 54
010 42   055 85   100 00   145 85   190 65
011 00   056 43   101 65   146 43   191 43
012 03   057 00   102 43   147 01   192 00
013 81   058 01   103 00   148 00   193 09
014 42   059 55   104 09   149 65   194 33
015 00   060 43   105 33   150 43   195 20
016 04   061 00   106 65   151 00   196 95
017 81   062 02   107 43   152 09   197 55
018 42   063 54   108 00   153 32   198 02
019 00   064 42   109 09   154 40   199 95
020 05   065 01   110 32   155 55   200 42
021 81   066 05   111 85   156 53   201 01
022 42   067 95   112 43   157 43   202 04
023 00   068 42   113 00   158 01   203 81
024 06   069 01   114 09   159 01
025 81   070 00   115 32   160 65
026 42   071 81   116 54   161 43
027 00   072 43   117 55   162 00
028 07   073 00   118 53   163 09
029 81   074 01   119 43   164 33
030 42   075 55   120 01   165 40
031 00   076 43   121 01   166 85
032 08   077 00   122 65   167 43
033 81   078 03   123 43   168 00
034 46   079 95   124 00   169 09
035 12   080 55   125 09   170 32
036 01   081 43   126 33   171 40
037 75   082 01   127 40   172 54
038 43   083 05   128 85   173 42
039 00   084 95   129 43   174 01
040 01   085 42   130 00   175 06
041 55   086 01   131 09   176 85
042 43   087 01   132 32   177 53
043 00   088 81   133 40   178 43
044 02   089 46   134 54   179 00
```

TITLE: STRENGTH PREDICTION (FAILURE POTENTIAL) NO: 123

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A Data | B $x_o$, etc. | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $\hat{t}$ | 02 L | 03 $C_1$ | 04 $\beta$ | 05 $\alpha$ | 06 $C_2$ |
| 07 $\epsilon_3$ | 08 $x_o$ | 09 $\Gamma(\frac{1}{\alpha(\beta+1)}+1)$ | 10 $\Gamma(\frac{2}{\alpha(\beta+1)}+1)$ | | |
| 11 $\gamma$ | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 123 A&B | $\hat{t}$ | A | $\hat{t}$ |
| | | | $\beta$ | RUN | $\beta$ |
| | | | $C_1$ | RUN | $C_1$ |
| | | | $\alpha$ | RUN | $\alpha$ |
| | | | $C_3$ | RUN | $C_3$ |
| | | | $C_2$ | RUN | $C_2$ |
| | | | L | RUN | L |
| | | | $\gamma$(0.5772) | RUN | $\gamma$ |
| Compute | 2 | | | B | $x_o$ |
| | | | | RUN | $1+1/\alpha(\beta+1)$ |
| | | | $\Gamma(1+1/\alpha(\beta+1))$ | RUN | $1+2/\alpha(\beta+1)$ |
| | | | $\Gamma(1+2/\alpha(\beta+1))$ | RUN | $\bar{x}$ (power) |
| | | | | RUN | c.v. (power) |
| | | | | C | $\bar{x}$ (exp.) |

PROGRAM TITLE: STRENGTH PREDICTION (FAILURE POTENTIAL) NO: 123

FORMULAS:

$$\bar{x}\,(\text{power}) = x_o \, \Gamma\left[1 + \frac{1}{\alpha(\beta + 1)}\right]$$

$$x_o = Lt_o = \left[\hat{t}\, L\, C_1{}^{\beta}(\beta+1)\right]^{1/(\beta+1)} \qquad (580)$$

$$\text{C. V.} = \left[\frac{\Gamma[1 + 2/\alpha(\beta + 1)]}{\Gamma^2[1 + 1/\alpha(\beta + 1)]} - 1\right]^{1/2}$$

$$\bar{x} = \frac{C_3}{\alpha}\left[\alpha \ln(\hat{t}LC_2/C_3) - \gamma\right] \qquad \gamma\text{: Euler constant } (=0.5772) \qquad (585)$$

**PROGRAM TITLE:** STRENGTH PREDICTION (FAILURE POTENTIAL) **NO:** 123

**PROGRAM LISTING:**

```
000 46   045 43   090 43   135 09   180 54
001 11   046 00   091 00   136 40   181 95
002 42   047 04   092 04   137 75   182 81
003 00   048 85   093 85   138 01   183 00
004 01   049 01   094 01   139 95   184 00
005 81   050 54   095 54   140 30   185 00
006 42   051 95   096 95   141 95
007 00   052 45   097 20   142 81
008 04   053 53   098 42   143 46
009 81   054 43   099 01   144 13
010 42   055 00   100 02   145 43
011 00   056 04   101 85   146 00
012 03   057 85   102 01   147 07
013 81   058 01   103 95   148 55
014 42   059 54   104 81   149 43
015 00   060 20   105 42   150 00
016 05   061 95   106 00   151 05
017 81   062 65   107 09   152 65
018 42   063 43   108 43   153 53
019 00   064 00   109 01   154 43
020 07   065 03   110 02   155 00
021 81   066 45   111 65   156 05
022 42   067 53   112 02   157 65
023 00   068 43   113 85   158 53
024 06   069 00   114 01   159 43
025 81   070 04   115 95   160 00
026 42   071 55   116 81   161 01
027 00   072 53   117 42   162 65
028 02   073 43   118 01   163 43
029 81   074 00   119 00   164 00
030 42   075 04   120 43   165 02
031 01   076 85   121 00   166 65
032 01   077 01   122 08   167 43
033 81   078 54   123 65   168 00
034 46   079 54   124 43   169 06
035 12   080 95   125 00   170 55
036 43   081 42   126 09   171 43
037 00   082 00   127 95   172 00
038 01   083 08   128 81   173 07
039 65   084 81   129 43   174 54
040 43   085 43   130 01   175 23
041 00   086 00   131 00   176 75
042 02   087 05   132 55   177 43
043 65   088 65   133 43   178 01
044 53   089 53   134 00   179 01
```

|  |  |  |  |  |
|---|---|---|---|---|
| TITLE: RESIDUAL STRENGTH (a) |  |  |  | NO: 127 |
| A' Parameters | B' | C' | D' | E' |
| A $N_o$ | B $R_\ell$ | C $R_r$ | D | E |

## REGISTER

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 00 |  |  |  |  |  |
| 01 $\alpha_s$ | 02 $x_o$ | 03 $\alpha_r$ | 04 $\alpha_\ell$ | 05 $\beta$ | 06 $C_1$ |
| 07 $n$ | 08 $S_{max}$ | 09 $\hat{x}$ | 10 $N_o$ |  |  |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 |  |  |  |
| 98 | 99 |  |  |  |  |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter parameters | 1 | 127 A&B | $\alpha_S$ | A' | $\alpha_S$ |
|  |  |  | $x_o$ | RUN | $x_o$ |
|  |  |  | $\alpha_r$ | RUN | $\alpha_r$ |
|  |  |  | $\alpha_\ell$ | RUN | $\alpha_\ell$ |
|  |  |  | $\beta$ | RUN | $\beta$ |
|  |  |  | $C_1$ | RUN | $C_1$ |
|  |  |  | $\hat{x}$ | RUN | $\hat{x}$ |
| Compute $N_o$ | 2 |  | $S_{max}$ | A | $N_o$ |
| Compute $R_\ell$ | 3 |  | $n$ | B | $R_\ell$ |
| Compute $R_r$ | 4 |  | $n$ | C | $R_r$ |

PROGRAM TITLE: RESIDUAL STRENGTH (a) NO: 127

FORMULAS:

$$R_r(\bar{x}_r) = \exp\left[ - \frac{\bar{x}_r^{\alpha_s} + (\alpha_s/\alpha_\ell)(S_{max}/C_1)^{\beta} n^{\alpha_\ell}}{\bar{x}_o^{\alpha_s}} + \left(\frac{\bar{S}_{max}}{\bar{x}_o}\right)^{\alpha_s} \right]$$

$$= \exp\left[ - \left(\frac{\bar{x}_r}{\bar{x}_o}\right)^{\alpha_s} - \left(\frac{n}{N_o}\right)^{\alpha_\ell} + \left(\frac{S_{max}}{x_o}\right)^{\alpha_s} \right] \qquad (622)$$

$$R_\ell(N) = \exp\left[ - \left(\frac{N}{N_o}\right)^{\alpha_\ell} \right] \qquad (623)$$

$$N_o(S_{max}/C_1)^{\beta/\alpha_\ell} = (x_o^{\alpha_s} \alpha_\ell/\alpha_s)^{1/\alpha_\ell} \qquad (624)$$

**PROGRAM TITLE:** RESIDUAL STRENGTH (a) **NO:** 127

**PROGRAM LISTING:**

```
000 46  045 43  090 45  135 43
001 16  046 00  091 43  136 00
002 42  047 01  092 00  137 02
003 00  048 95  093 04  138 54
004 01  049 65  094 95  139 45
005 81  050 43  095 94  140 43
006 42  051 00  096 22  141 00
007 00  052 04  097 23  142 01
008 02  053 55  098 81  143 95
009 81  054 43  099 46  144 22
010 42  055 00  100 13  145 23
011 00  056 01  101 42  146 81
012 03  057 95  102 00
013 81  058 55  103 07
014 42  059 53  104 55
015 00  060 43  105 43
016 04  061 00  106 00
017 81  062 08  107 02
018 42  063 55  108 95
019 00  064 43  109 45
020 05  065 00  110 43
021 81  066 06  111 00
022 42  067 54  112 01
023 00  068 45  113 95
024 06  069 43  114 94
025 81  070 00  115 75
026 42  071 05  116 53
027 00  072 95  117 43
028 09  073 45  118 00
029 81  074 43  119 07
030 46  075 00  120 55
031 11  076 04  121 43
032 42  077 20  122 01
033 00  078 95  123 00
034 08  079 42  124 54
035 53  080 01  125 45
036 43  081 00  126 43
037 00  082 81  127 00
038 02  083 46  128 04
039 55  084 12  129 85
040 43  085 55  130 53
041 00  086 43  131 43
042 09  087 01  132 00
043 54  088 00  133 08
044 45  089 95  134 55
```

TITLE: RESIDUAL STRENGTH (b) NO: 131

| | | | | |
|---|---|---|---|---|
| A' Parameters | B' | C' | D' | E' |
| A $N_o$ | B $R_\ell$ | C $R_r$ | D | E |

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $\alpha_s$ | 02 $x_o$ | 03 $\alpha_r$ | 04 $\alpha_\ell$ | 05 $\beta$ | 06 $C_1$ |
| 07 $n$ | 08 $S_{max}$ | 09 $\hat{x}$ | 10 $N_o$ | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## REGISTER

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter parameters | 1 | 131 A&B | $\alpha_S$ | A' | |
| | | | $x_o$ | RUN | |
| | | | $\alpha_r$ | RUN | |
| | | | $\alpha_\ell$ | RUN | |
| | | | $\beta$ | RUN | |
| | | | $C_1$ | RUN | |
| | | | $\hat{x}$ | RUN | |
| Compute $N_o$ | 2 | | $S_{max}$ | A | $N_o$ |
| Compute $R_\ell$ | 3 | | $n$ | B | $R_\ell$ |
| Compute $R_r$ | 4 | | $n$ | C | $R_r$ |

**PROGRAM TITLE:** RESIDUAL STRENGTH (b) **NO:** 131

**FORMULAS:**

$$R_r(\bar{x}_r) = \exp\left[ - \frac{\left(\bar{x}_r^{\alpha_r} + \alpha_r (S_{max}/C_1)^{\beta} n\right)^{\alpha_s/\alpha_r}}{\bar{x}_o^{\alpha_s}} + \left(\frac{S_{max}}{x_o}\right)^{\alpha_s} \right]$$

$$= \exp\left\{ - \left[ \left(\frac{\bar{x}_r}{\bar{x}_o}\right)^{\alpha_r} + \left(\frac{n}{N_o}\right) \right]^{\alpha_s/\alpha_r} + \left(\frac{S_{max}}{x_o}\right)^{\alpha_s} \right\} \tag{627}$$

$$R_{\ell}(N) = \exp\left[ - \left(\frac{N}{N_o}\right)^{\alpha_s/\alpha_r} \right] \tag{628}$$

$$N_o (S_{max}/C_1)^{\beta} = \bar{x}_o^{\alpha_r}/\alpha_r \tag{629}$$

**PROGRAM TITLE:** RESIDUAL STRENGTH (b) **NO:** 131

**PROGRAM LISTING:**

```
000 46   045 20   090 23   135 02
001 16   046 65   091 81   136 54
002 42   047 53   092 46   137 45
003 00   048 43   093 13   138 43
004 01   049 00   094 42   139 00
005 81   050 02   095 00   140 01
006 42   051 55   096 07   141 95
007 00   052 43   097 55   142 22
008 02   053 00   098 43   143 23
009 81   054 09   099 00   144 81
010 42   055 54   100 02
011 00   056 45   101 95
012 03   057 43   102 45
013 81   058 00   103 43
014 42   059 03   104 00
015 00   060 95   105 03
016 04   061 55   106 85
017 81   062 43   107 43
018 42   063 00   108 00
019 00   064 03   109 07
020 05   065 95   110 55
021 81   066 42   111 43
022 42   067 01   112 01
023 00   068 00   113 00
024 06   069 81   114 95
025 81   070 46   115 45
026 42   071 12   116 53
027 00   072 55   117 43
028 09   073 43   118 00
029 81   074 01   119 01
030 46   075 00   120 55
031 11   076 95   121 43
032 42   077 45   122 00
033 00   078 53   123 03
034 08   079 43   124 54
035 55   080 00   125 95
036 43   081 01   126 94
037 00   082 55   127 85
038 06   083 43   128 53
039 95   084 00   129 43
040 45   085 03   130 00
041 43   086 54   131 08
042 00   087 95   132 55
043 05   088 94   133 43
044 95   089 22   134 00
```

| TITLE: CUMULATIVE DAMAGE | | | | NO: 135 |
|---|---|---|---|---|
| A' Data | B' | C' | D' | E' |
| A $R_{\ell n_1}$ | B $R_{\ell n_1}$ | C | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $N_{o1}$ | 02 $N_{o2}$ | 03 $n_1$ | 04 $\alpha_\ell$ | 05 $\alpha_s$ | 06 $\alpha_r$ |
| 07 | 08 | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 135 A | $N_{o1}$ | A' | $N_{o1}$ |
| | | | $N_{o2}$ | RUN | $N_{o2}$ |
| | | | $n_1$ | RUN | $n_1$ |
| | | | $\alpha_\ell$ | RUN | $\alpha_\ell$ |
| | | | $\alpha_s$ | RUN | $\alpha_s$ |
| | | | $\alpha_r$ | RUN | $\alpha_r$ |
| Compute $R_{\ell n_1}$ model (a) | 2 | | $N_2$ | A | $R_{\ell n_1}$ |
| model (b) | | | $N_2$ | B | $R_{\ell n_1}$ |

PROGRAM TITLE: CUMULATIVE DAMAGE NO: 135

FORMULAS:

(a) $\alpha_r = \alpha_s$

$$R_{\ell n1}(N_2) = \exp\left[-\left(\frac{n_1}{N_{o1}}\right)^{\alpha_\ell} - \left(\frac{N_2}{N_{o2}}\right)^{\alpha_\ell}\right] \qquad (634)$$

(b) $\alpha_\ell = 1$

$$R_{\ell n1}(N_2) = \exp\left[-\left(\frac{n_1}{N_{o1}} + \frac{N_2}{N_{o2}}\right)^{\alpha_s/\alpha_r}\right] \qquad (636)$$

**PROGRAM TITLE:** CUMULATIVE DAMAGE **NO:** 135

**PROGRAM LISTING:**

```
000 46   045 00
001 16   046 01
002 42   047 54
003 00   048 45
004 01   049 43
005 81   050 00
006 42   051 04
007 00   052 95
008 02   053 94
009 81   054 22
010 42   055 23
011 00   056 95
012 03   057 81
013 81   058 46
014 42   059 12
015 00   060 55
016 04   061 43
017 81   062 00
018 42   063 02
019 00   064 85
020 05   065 43
021 81   066 00
022 42   067 03
023 00   068 55
024 06   069 43
025 81   070 00
026 46   071 01
027 11   072 95
028 55   073 45
029 43   074 53
030 00   075 43
031 02   076 00
032 95   077 05
033 45   078 55
034 43   079 43
035 00   080 00
036 04   081 06
037 95   082 54
038 85   083 95
039 53   084 94
040 43   085 22
041 00   086 23
042 03   087 95
043 55   088 81
044 43
```

TITLE: COMPOSITE CONDUCTIVITIES NO: 139

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $K_{11}$ | B $K_{22}$ | C $K_x$ | D | E |

REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 $\alpha$ | | | | | |
| 01 $K_m$ | 02 $K_f$ | 03 $v_f$ | 04 $B_k$ | 05 $B_k^2 v_f/\pi$ | 06 $K_{22}$ |
| 07 $K_{11}$ | 08 $K_x$ | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input data | 1 | 139 A&B | $K_m$ | A | $K_m$ |
| | | | $K_f$ | RUN | $K_f$ |
| | | | $v_f$ | RUN | $K_{11}$ |
| | | | | B | $K_{22}$ |
| | | | $\alpha$ | C | $K_x$ |

PROGRAM TITLE: COMPOSITE CONDUCTIVITIES NO: 139

FORMULAS:

Heat Transfer (temperature effects)

$$q = -K\nabla T \tag{638}$$

One dimensional steady

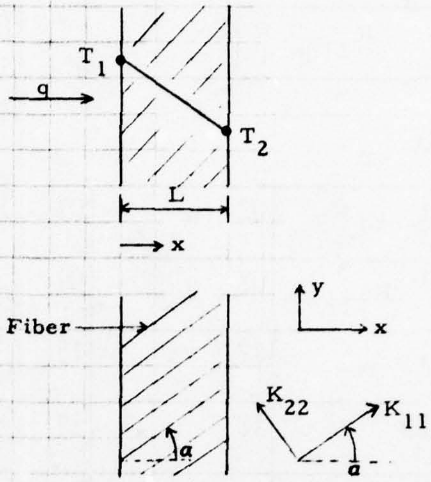
$$q_x = +K_x \frac{T_1 - T_2}{L} \tag{639}$$

Composite Materials

$$K_x = K_{11}\cos^2 a + K_{22}\sin^2 a \tag{640}$$

Approximations ($v_f < 0.785$)

$$K_{11} = (1 - v_f)K_m + v_f K_f \tag{641}$$



$$K_{22} = K_m\left\{(1 - 2\sqrt{v_f/\pi}) + \frac{1}{B_K}\left[\pi - \frac{4}{\sqrt{1 - B_K^2 v_f/\pi}} \tan^{-1}\frac{\sqrt{1 - (B_K^2 v_f/\pi)}}{1 + B_K(v_f/\pi)^{1/2}}\right]\right\} \tag{642}$$

$$B_K \equiv 2\left(\frac{K_m}{K_f} - 1\right) \tag{643}$$

PROGRAM TITLE: COMPOSITE CONDUCTIVITIES NO: 139

PROGRAM LISTING:

```
000 46   045 02   090 20   135 81
001 11   046 75   091 22   136 46
002 42   047 01   092 34   137 13
003 00   048 95   093 65   138 42
004 01   049 65   094 04   139 00
005 81   050 02   095 55   140 00
006 42   051 95   096 53   141 33
007 00   052 42   097 01   142 40
008 02   053 00   098 75   143 65
009 81   054 04   099 43   144 43
010 42   055 40   100 00   145 00
011 00   056 65   101 05   146 07
012 03   057 43   102 54   147 85
013 43   058 00   103 30   148 43
014 00   059 03   104 95   149 00
015 01   060 55   105 75   150 06
016 65   061 59   106 59   151 65
017 53   062 95   107 95   152 43
018 01   063 42   108 94   153 00
019 75   064 00   109 55   154 00
020 43   065 05   110 43   155 32
021 00   066 43   111 00   156 40
022 03   067 00   112 04   157 95
023 54   068 03   113 85   158 42
024 85   069 55   114 01   159 00
025 43   070 59   115 75   160 08
026 00   071 54   116 02   161 81
027 03   072 30   117 65
028 65   073 65   118 53
029 43   074 43   119 43
030 00   075 00   120 00
031 02   076 04   121 03
032 95   077 85   122 55
033 42   078 01   123 59
034 00   079 54   124 54
035 07   080 55   125 30
036 81   081 53   126 95
037 46   082 01   127 65
038 12   083 75   128 43
039 43   084 43   129 00
040 00   085 00   130 01
041 01   086 05   131 95
042 55   087 54   132 42
043 43   088 30   133 00
044 00   089 95   134 06
```

|  |  |  |  |  |
|---|---|---|---|---|
| TITLE: MOISTURE GAIN AND DISTRIBUTION |  |  | NO: 143 |  |
| A' | B' | C' t | D' | E' |
| A $D_x$, S, h, $x^*$ | B | C $t^*$ | D $C^*$ | E G |

## REGISTER

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 00 |  |  |  |  |  |
| 01 $D_x$ | 02 $D_x/S^2$ | 03 h | 04 $x^*=x/h$ | 05 $t^*=D_xt/S^2$ | 06 (2j+1) |
| 07 SUM G | 08 SUM $C^*$ | 09 exp[ ] | 10 G |  |  |
| 11 $C^*$ | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 |  |  |  |
| 98 | 99 |  |  |  |  |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Input Data | 1 | 143 A&B | $D_x$ | A | $D_x$ |
|  |  |  | S | RUN | $D_x/S^2$ |
|  |  |  | h | RUN | h |
|  |  |  | x | RUN | $x^*=x/h$ |
|  |  |  | t | RUN | $t^*=D_xt/S^2$ |
| Compute $C^*$ and G | 2 |  | j=0 | B |  |
|  |  |  | 1 | B |  |
|  |  |  | 2 | B |  |
|  |  |  | 3 | B |  |
|  |  |  | . | . |  |
|  |  |  | . | . |  |
|  |  |  | . | . | G |
|  |  |  |  | C | $C^*$ |
|  |  |  |  | D | t |

**PROGRAM TITLE:** MOISTURE GAIN AND DISTRIBUTION **NO:** 143

**FORMULAS:**

Moisture Effects

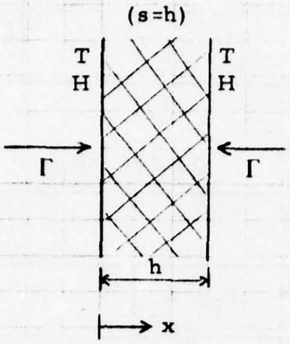
$$\Gamma = -D\nabla C \qquad (644)$$

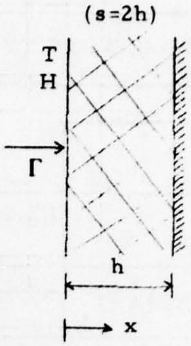
Steady Bound. Cond.

One Dimensional

Temp Constant

Uniform Initial Distributions





Moisture Distribution:

$$C^{*} = \frac{C-C_i}{C_m-C_i} = 1 - \frac{4}{\pi}\sum_{j=0}^{\infty}\frac{1}{(2j+1)}\sin\frac{(2j+1)\pi x}{h}\exp\left\{-\left[(2j+1)^2\pi^2\right]\frac{D_x t}{s^2}\right\} \qquad (645)$$

Moisture Content (weight gain):

$$M = \frac{W-W_D}{W_D} = G(M_m - M_i) + M_i \qquad (646)$$

$$G \equiv 1 - \frac{8}{\pi^2}\sum_{j=0}^{\infty}\frac{\exp\left[-(2j+1)^2\pi^2\left(\frac{D_x t}{s^2}\right)\right]}{(2j+1)^2} \qquad (647)$$

Time to reach 99.9% saturation:

$$t_m = \frac{0.67\, s^2}{D_x} \qquad (648)$$

**PROGRAM TITLE:** MOISTURE GAIN AND DISTRIBUTION **NO:** 143

**PROGRAM LISTING:**

```
045 07   000 46   090 59   135 54
046 42   001 11   091 40   136 10
047 00   002 42   092 75   137 30
048 08   003 00   093 01   138 14
049 81   004 01   094 95   139 00
050 46   005 81   095 94   140 10
051 12   006 40   096 42   141 68
052 65   007 55   097 01   142 34
053 02   008 43   098 00   143 41
054 85   009 00   099 43   144 14
055 01   010 01   100 00   145 10
056 95   011 95   101 06   146 10
057 42   012 20   102 65   147 68
058 00   013 42   103 43   148 44
059 06   014 00   104 00   149 31
060 40   015 02   105 04   150 04
061 65   016 81   106 65   151 10
062 59   017 42   107 59   152 50
063 40   018 00   108 95   153 35
064 65   019 03   109 22   154 04
065 43   020 81   110 38   155 20
066 00   021 55   111 32   156 50
067 05   022 43   112 36   157 19
068 95   023 00   113 04   158 08
069 94   024 03   114 92   159 30
070 22   025 95   115 38
071 23   026 42   116 34
072 42   027 00   117 52
073 00   028 04   118 00
074 09   029 00   119 14
075 55   030 42   120 49
076 43   031 00   121 04
077 00   032 08   122 80
078 06   033 81   123 30
079 40   034 65   124 04
080 95   035 43   125 80
081 44   036 00   126 50
082 00   037 02   127 56
083 07   038 95   128 50
084 43   039 42   129 95
085 00   040 00   130 55
086 07   041 05   131 17
087 65   042 00   132 50
088 08   043 42   133 49
089 55   044 00   134 29
```

|  |  |  |  |  |
|---|---|---|---|---|
| TITLE: BEST FIT FOR WEIBULL DISTRIBUTION |  |  |  | NO: 147 |
| A' | B' | C' | D' | E' |
| A $x$ | B $j$ | C $r^2$(C.D.) | D $\alpha$ | E $x_o$ |

REGISTER

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 00 |  |  |  |  |  |
| 01 | 02 | 03 | 04 | 05 | 06 |
| 07 | 08 | 09 | 10 |  |  |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 |  |  |  |
| 98 | 99 |  |  |  |  |

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Initialize | 1 | 147 A&B |  | $CM_s$ |  |
|  |  |  | N | STO |  |
|  |  |  |  | 1 |  |
|  |  |  |  | 4 | N |
| Enter data | 2 |  | $x$ | A | lnx |
| Repeat for all data |  |  | $j$ | B | $j$ |
| Compute | 3 |  |  | C | $\alpha$ |
|  |  |  |  | RUN | $x_o$ |
|  |  |  |  | E | $r^2$(C.D.) |

PROGRAM TITLE: BEST FIT FOR WEIBULL DISTRIBUTION NO: 147

FORMULAS:

$$F = 1 - \exp\left[-(x/x_o)^{\alpha}\right] \qquad (702)$$

$$F = \frac{j - 0.3}{N + 0.4}$$

**PROGRAM TITLE:** BEST FIT FOR WEIBULL DISTRIBUTION **NO:** 147

**PROGRAM LISTING:**

```
000 46   045 02   090 43   135 00   180 41
001 88   046 44   091 00   136 04   181 87
002 44   047 00   092 04   137 95   182 46
003 00   048 04   093 75   138 55   183 13
004 00   049 40   094 43   139 43   184 14
005 43   050 44   095 00   140 00   185 65
006 00   051 00   096 05   141 00   186 43
007 00   052 07   097 95   142 95   187 01
008 75   053 43   098 94   143 94   188 03
009 01   054 00   099 42   144 55   189 55
010 95   055 02   100 01   145 43   190 53
011 30   056 65   101 03   146 01   191 43
012 42   057 43   102 55   147 01   192 00
013 00   058 00   103 53   148 95   193 04
014 08   059 01   104 43   149 22   194 40
015 25   060 95   105 00   150 23   195 55
016 43   061 44   106 06   151 56   196 43
017 00   062 00   107 75   152 46   197 00
018 00   063 05   108 43   153 16   198 00
019 56   064 43   109 00   154 51   199 94
020 46   065 00   110 03   155 77   200 85
021 12   066 01   111 65   156 94   201 43
022 75   067 44   112 43   157 85   202 00
023 93   068 00   113 01   158 43   203 07
024 03   069 03   114 00   159 01   204 95
025 95   070 40   115 95   160 02   205 56
026 55   071 44   116 42   161 95   206 46
027 53   072 00   117 01   162 55   207 11
028 43   073 06   118 01   163 46   208 23
029 01   074 01   119 56   164 87   209 42
030 04   075 41   120 46   165 43   210 00
031 85   076 88   121 77   166 01   211 01
032 93   077 46   122 42   167 01   212 56
033 04   078 14   123 01   168 95
034 54   079 43   124 02   169 56
035 95   080 00   125 46   170 46
036 94   081 03   126 15   171 17
037 85   082 55   127 14   172 51
038 01   083 43   128 65   173 77
039 95   084 00   129 43   174 85
040 23   085 00   130 00   175 53
041 94   086 65   131 03   176 43
042 23   087 42   132 94   177 01
043 42   088 01   133 85   178 02
044 00   089 00   134 43   179 65
```

TITLE: A AND B ALLOWABLES (WEIBULL) NO: 151

| | | | | |
|---|---|---|---|---|
| A' Parameters | B' | C' | D' | E' |
| A $x_A$ | B $x_B$ | C i + 1 | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $\alpha$ | 02 n | 03 $X^2_{2n;\gamma}$ | 04 j | 05 0.99 | 06 0.90 |
| 07 $\sum x_i^{\alpha}$ | 08 | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter parameters | 1 | 151 A | | A' | |
| | | | $\alpha$ | RUN | $\alpha$ |
| | | | n | RUN | n |
| | | | $X^2_{2n;\gamma}$ | RUN | $X^2_{2n;\gamma}$ |
| Enter data | 2 | | | C | |
| Repeat for all data | | | $x_i$ | RUN | i + 1 |
| Compute | 3 | | | A | $x_A$ |
| | | | | B | $x_B$ |

PROGRAM TITLE: A AND B ALLOWABLES (WEIBULL) NO: 151

FORMULAS:

For "A" allowable $x_A$ $\hat{R}_\gamma = 0.99$ , $\gamma = 0.95$

For "B" allowable $x_B$ $\hat{R}_\gamma = 0.90$ , $\gamma = 0.95$

$$x_{A,B} = [-2n\ell n\hat{R}_\gamma / X^2_{2n;\gamma}]^{1/\alpha}\hat{x}_o \qquad (728)$$

$$\hat{x}_o = [\frac{1}{n}\sum_{i=1}^{n} x_i^\alpha]^{1/\alpha} \qquad (729)$$

**PROGRAM TITLE:** A AND B ALLOWABLES (WEIBULL) **NO:** 151

**PROGRAM LISTING:**

```
000 46   045 46   090 65   135 03   180 43
001 16   046 11   091 43   136 94   181 01
002 81   047 43   092 00   137 85   182 02
003 42   048 00   093 02   138 43   183 65
004 00   049 07   094 65   139 00   184 41
005 01   050 55   095 93   140 04   185 87
006 81   051 43   096 09   141 95   186 46
007 42   052 00   097 00   142 55   187 13
008 00   053 04   098 23   143 43   188 14
009 02   054 65   099 95   144 00   189 65
010 81   055 02   100 55   145 00   190 43
011 42   056 65   101 43   146 95   191 01
012 00   057 43   102 00   147 94   192 03
013 03   058 00   103 03   148 55   193 55
014 81   059 02   104 95   149 43   194 53
015 46   060 65   105 94   150 01   195 43
016 13   061 93   106 45   151 01   196 00
017 00   062 09   107 43   152 95   197 04
018 42   063 09   108 00   153 22   198 40
019 00   064 23   109 01   154 23   199 55
020 04   065 95   110 20   155 56   200 43
021 42   066 55   111 95   156 46   201 00
022 00   067 43   112 81   157 16   202 00
023 07   068 00   113 00   158 51   203 94
024 01   069 03   114 03   159 77   204 85
025 44   070 95   115 65   160 94   205 43
026 00   071 94   116 43   161 85   206 00
027 04   072 45   117 01   162 43   207 07
028 43   073 43   118 00   163 01   208 95
029 00   074 00   119 95   164 02   209 56
030 04   075 01   120 42   165 95   210 46
031 81   076 20   121 01   166 55   211 11
032 45   077 95   122 01   167 46   212 23
033 43   078 81   123 56   168 87   213 42
034 00   079 46   124 46   169 43   214 00
035 01   080 12   125 77   170 01   215 01
036 95   081 43   126 42   171 01   216 56
037 44   082 00   127 01   172 95
038 00   083 07   128 02   173 56
039 07   084 55   129 46   174 46
040 41   085 43   130 15   175 17
041 00   086 00   131 14   176 51
042 02   087 04   132 65   177 77
043 04   088 65   133 43   178 85
044 81   089 02   134 00   179 53
```

TITLE: SIZE EFFECT (a) NO: 155

| | | | | |
|---|---|---|---|---|
| A' Dimensions | B' Dimensions | C' | D' | E' |
| A $V_t$, $A_t$, $\ell_t$ | B $V_f$, $A_f$, $\ell_f$ | C Volume, 3-pt | D Volume, 4-pt | E |

## REGISTER

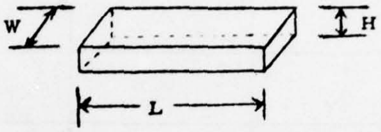
| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $W_t$ | 02 $L_t$ | 03 $H_t$ | 04 $W_f$ | 05 $L_f$ | 06 $H_f$ |
| 07 $V_t$ | 08 $A_t$ | 09 $\ell_t$ | 10 $V_f$ | | |
| 11 $A_f$ | 12 $B_f$ | 13 $\ell_f$ | 14 $\alpha$ | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter dimensions | 1 | 155 A&B | | A' | |
| | | | $W_t$ | RUN | $W_t$ |
| | | | $L_t$ | RUN | $L_t$ |
| | | | $H_t$ | RUN | $H_t$ |
| | | | $W_f$ | B' | $W_f$ |
| | | | $L_f$ | B' | $L_f$ |
| | | | $H_f$ | B' | $H_f$ |
| Compute | 2 | | | A | $V_t$ |
| | | | | RUN | $A_t$ |
| | | | | RUN | $\ell_t$ |
| | | | | B | $V_f$ |
| | | | | RUN | $A_f$ |
| | | | | RUN | $\ell_f$ |
| Compute (volume model) | 3 | | | C | |
| | | | $\alpha$ | RUN | $X_{of}/X_{ot}$ (3-pt |
| | | | | D | |
| | | | $\alpha$ | RUN | $X_{of}/X_{ot}$ (4-pt) |

PROGRAM TITLE: SIZE EFFECT (a) NO: 155

FORMULAS:

<table>
<tr><th>3-pt. flexure<br>(center point loading)</th><th>4-pt. flexure<br>(quarter point loading)</th></tr>
<tr><td>Volume $\frac{(X_o)_f}{(X_o)_t} = \left[2(\alpha+1)^2 \frac{V_t}{V_f}\right]^{\frac{1}{\alpha}}$</td><td>$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{4(\alpha+1)^2}{\alpha+2} \frac{V_t}{V_f}\right]^{\frac{1}{\alpha}}$</td></tr>
<tr><td colspan="2">$V_t = WLH$, $V_f = WLH$<br>$A_t = 2L(W+H)$, $A_f = WL$, $B_f = LH$<br>$\ell_t = 4L$, $\ell_f = 2L$<br> </td></tr>
</table>

**PROGRAM TITLE:** SIZE EFFECT (a) **NO:** 155

**PROGRAM LISTING:**

```
000 46   045 00   090 06   135 04   180 00
001 16   046 03   091 95   136 85   181 07
002 81   047 54   092 42   137 01   182 55
003 42   048 95   093 01   138 95   183 43
004 00   049 42   094 00   139 40   184 01
005 01   050 00   095 81   140 65   185 00
006 81   051 08   096 43   141 02   186 95
007 42   052 81   097 00   142 65   187 45
008 00   053 04   098 04   143 43   188 43
009 02   054 65   099 65   144 00   189 01
010 81   055 43   100 43   145 07   190 04
011 42   056 00   101 00   146 55   191 20
012 00   057 02   102 05   147 43   192 95
013 03   058 95   103 95   148 01   193 81
014 81   059 42   104 42   149 00   194 53
015 46   060 00   105 01   150 95   195 43
016 11   061 09   106 01   151 45   196 00
017 43   062 81   107 81   152 43   197 04
018 00   063 46   108 43   153 01   198 40
019 01   064 17   109 00   154 04   199 55
020 65   065 81   110 05   155 20   200 43
021 43   066 42   111 65   156 95   201 00
022 00   067 00   112 43   157 81   202 00
023 02   068 04   113 00   158 46   203 94
024 65   069 81   114 06   159 14   204 85
025 43   070 42   115 95   160 81   205 43
026 00   071 00   116 42   161 42   206 00
027 03   072 05   117 01   162 01   207 07
028 95   073 81   118 02   163 04   208 95
029 42   074 42   119 81   164 85   209 56
030 00   075 00   120 43   165 01   210 46
031 07   076 06   121 00   166 95   211 11
032 81   077 81   122 05   167 40   212 23
033 43   078 46   123 65   168 65   213 42
034 00   079 12   124 02   169 04   214 00
035 02   080 43   125 95   170 55   215 01
036 65   081 00   126 42   171 53   216 56
037 02   082 04   127 01   172 43
038 65   083 65   128 03   173 01
039 53   084 43   129 81   174 04
040 43   085 00   130 46   175 85
041 00   086 05   131 13   176 02
042 01   087 65   132 81   177 54
043 85   088 43   133 42   178 65
044 43   089 00   134 01   179 43
```

TITLE: SIZE EFFECT (b) NO: 159

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $X_{of}/X_{ot}$ | B Surface, 3-pt | C Surface, 4-pt | D Edge, 3-pt | E Edge, 4-pt |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $W_t$ | 02 $L_t$ | 03 $H_t$ | 04 $W_f$ | 05 $L_f$ | 06 $H_f$ |
| 07 $V_t$ | 08 $A_t$ | 09 $\ell_t$ | 10 $V_f$ | | |
| 11 $A_f$ | 12 $B_f$ | 13 $\ell_f$ | 14 $\alpha$ | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Compute dimensions with 155 | 1 | | | | |
| Compute | 2 | 159 A&B | | | |
| Surface, 3-pt | | | | A | |
| | | | $\alpha$ | RUN | $X_{of}/X_{ot}$ |
| Surface, 4-pt | | | | B | |
| | | | $\alpha$ | RUN | $X_{of}/X_{ot}$ |
| Edge, 3-pt | | | | C | |
| | | | $\alpha$ | RUN | $X_{of}/X_{ot}$ |
| Edge, 4-pt | | | | D | |
| | | | $\alpha$ | RUN | $X_{of}/X_{ot}$ |

PROGRAM TITLE: SIZE EFFECT (b) NO: 159

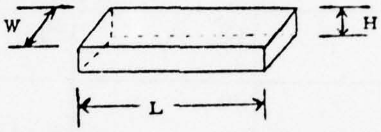
FORMULAS:

| | 3-pt. flexure<br>(center point loading) | 4-pt. flexure<br>(quarter point loading) |
|---|---|---|
| **Surface** | $\frac{(X_o)_f}{(X_o)_t} = \left[\frac{(\alpha+1)A_t}{A_f + B_f/(\alpha+1)}\right]^{\frac{1}{\alpha}}$ | $\frac{(X_o)_f}{(X_o)_t} = \left[\frac{(\alpha+1)^2 A_t}{B_f/2 + (\alpha+1)(A_f + B_f)/2 + (\alpha+1)^2 A_f/2}\right]^{\frac{1}{\alpha}}$ |
| **Edge** | $\frac{(X_o)_f}{(X_o)_t} = \left[(\alpha+1)\frac{\ell_t}{\ell_f}\right]^{\frac{1}{\alpha}}$ | $\frac{(X_o)_f}{(X_o)_t} = \left[\frac{\alpha+1}{\alpha+2}\,\frac{\ell_t}{\ell_f/2}\right]^{\frac{1}{\alpha}}$ |

$V_t = WLH$, $V_f = WLH$

$A_t = 2L(W+H)$, $A_f = WL$, $B_f = LH$

$\ell_t = 4L$, $\ell_f = 2L$



**PROGRAM TITLE:** SIZE EFFECT (b) **NO:** 159

**PROGRAM LISTING:**

```
000 46   045 85   090 43   135 85   180 00
001 11   046 01   091 01   136 01   181 07
002 81   047 95   092 01   137 95   182 55
003 42   048 40   093 55   138 55   183 43
004 01   049 65   094 02   139 53   184 01
005 04   050 43   095 54   140 43   185 00
006 85   051 00   096 95   141 01   186 95
007 01   052 08   097 45   142 04   187 45
008 95   053 55   098 43   143 85   188 43
009 65   054 53   099 01   144 02   189 01
010 43   055 43   100 04   145 54   190 04
011 00   056 01   101 20   146 65   191 20
012 08   057 02   102 95   147 43   192 95
013 55   058 55   103 81   148 00   193 81
014 53   059 02   104 46   149 09   194 53
015 43   060 85   105 13   150 55   195 43
016 01   061 53   106 81   151 43   196 00
017 01   062 43   107 42   152 01   197 04
018 85   063 01   108 01   153 03   198 40
019 43   064 04   109 04   154 65   199 55
020 01   065 85   110 85   155 02   200 43
021 02   066 01   111 01   156 95   201 00
022 55   067 54   112 95   157 45   202 00
023 53   068 65   113 65   158 43   203 94
024 43   069 53   114 43   159 01   204 85
025 01   070 43   115 00   160 04   205 43
026 04   071 01   116 09   161 20   206 00
027 85   072 01   117 55   162 95   207 07
028 01   073 85   118 43   163 81   208 95
029 54   074 43   119 01   164 85   209 56
030 54   075 01   120 03   165 01   210 46
031 95   076 02   121 95   166 95   211 11
032 45   077 54   122 45   167 40   212 23
033 43   078 55   123 43   168 65   213 42
034 01   079 02   124 01   169 04   214 00
035 04   080 85   125 04   170 55   215 01
036 20   081 53   126 20   171 53   216 56
037 95   082 43   127 95   172 43
038 81   083 01   128 81   173 01
039 46   084 04   129 46   174 04
040 12   085 85   130 14   175 85
041 81   086 01   131 81   176 02
042 42   087 54   132 42   177 54
043 01   088 40   133 01   178 65
044 04   089 65   134 04   179 43
```

| TITLE: DATA AVERAGING (a) | | | | NO: 163 |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $\sigma_2$ | B $\sigma_6$ | C $A_{ij}$, $B_i$ | D | E |

## REGISTER

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $\sigma_2$ | 02 $\sigma_2^2$ | 03 $\sigma_6$ | 04 | 05 | 06 |
| 07 | 08 | 09 | 10 $\Sigma\sigma_2^2$ | | |
| 11 $\Sigma\sigma_2^3$ | 12 $\Sigma\sigma_2\sigma_6^2$ | 13 $\Sigma\sigma_2^4$ | 14 $\Sigma\sigma_2^2\sigma_6^2$ | 15 $\Sigma\sigma_6^4$ | 16 $\Sigma\sigma_2$ |
| 17 $\Sigma\sigma_2^2$ | 18 $\Sigma\sigma_6^2$ | 19 | | | |
| 98 | 99 | | | | |

## INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 163 A&B | $\sigma_2$ | A | $\Sigma\sigma_2$ |
| Repeat for all data | | | $\sigma_6$ | B | $\Sigma\sigma_6$ |
| Display results | 2 | | | C | $A_{11}$ |
| | | | | RUN | $A_{12}$ |
| | | | | RUN | $A_{13}$ |
| | | | | RUN | $A_{22}$ |
| | | | | RUN | $A_{23}$ |
| | | | | RUN | $A_{33}$ |
| | | | | RUN | $B_1$ |
| | | | | RUN | $B_2$ |
| | | | | RUN | $B_3$ |

PROGRAM TITLE: DATA AVERAGING (a) NO: 163

FORMULAS:

$$[A] = [\sigma]^T [\sigma]$$

cf. Eq. (769)

$$\{B\} = [\sigma]^T \{1\}$$

PROGRAM LISTING:

```
000 46        030 42        060 01        090 01
001 11        031 00        061 05        091 81
002 42        032 03        062 43        092 43
003 00        033 40        063 00        093 01
004 01        034 65        064 01        094 02
005 40        035 43        065 44        095 81
006 42        036 00        066 01        096 43
007 00        037 01        067 06        097 01
008 02        038 95        068 43        098 03
009 44        039 44        069 00        099 81
010 01        040 01        070 02        100 43
011 00        041 02        071 44        101 01
012 65        042 43        072 01        102 04
013 43        043 00        073 07        103 81
014 00        044 02        074 43        104 43
015 01        045 65        075 00        105 01
016 95        046 43        076 03        106 05
017 44        047 00        077 40        107 81
018 01        048 03        078 44        108 43
019 01        049 40        079 01        109 01
020 43        050 95        080 08        110 06
021 00        051 44        081 81        111 81
022 02        052 01        082 46        112 43
023 40        053 04        083 13        113 01
024 44        054 43        084 43        114 07
025 01        055 00        085 01        115 81
026 03        056 03        086 00        116 43
027 81        057 40        087 81        117 01
028 46        058 40        088 43        118 08
029 12        059 44        089 01        119 81
```

TITLE: DATA AVERAGING (b) NO: 165

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| A $F_{ij}$ | B $f_m$ | C | D | E |

| | | | | | |
|---|---|---|---|---|---|
| 00 | | | | | |
| 01 $F_2$ | 02 $F_{22}$ | 03 $F_{66}$ | 04 $\sigma_2$ | 05 | 06 |
| 07 | 08 | 09 | 10 | | |
| 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | | | |
| 98 | 99 | | | | |

REGISTER

INSTRUCTIONS

| OPERATION | STEP | READ | ENTER | PRESS | DISPLAY |
|---|---|---|---|---|---|
| Enter data | 1 | 165 A | | A | |
| | | | $F_2$ | RUN | $F_2$ |
| | | | $F_{22}$ | RUN | $F_{22}$ |
| | | | $F_{66}$ | RUN | $F_{66}$ |
| Compute | 2 | | $\sigma_2$ | B | $F_2\sigma_2+F_{22}\sigma_2^2$ |
| | | | $\sigma_6$ | RUN | $f_n$ |

PROGRAM TITLE: DATA AVERAGING (b) NO: 165

FORMULAS:

$$f_m(\sigma_2, \sigma_6) = F_2\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 . \qquad (775)$$

PROGRAM LISTING:

```
000 46    020 65    040 95    060 01    080 08
001 11    021 43    041 81    061 05    081 81
002 81    022 00    042 43    062 43
003 42    023 01    043 00    063 00
004 00    024 85    044 02    064 01
005 01    025 43    045 65    065 44
006 81    026 00    046 43    066 01
007 42    027 02    047 00    067 06
008 00    028 65    048 03    068 43
009 02    029 43    049 40    069 00
010 81    030 00    050 95    070 02
011 42    031 04    051 44    071 44
012 00    032 40    052 01    072 01
013 03    033 85    053 04    073 07
014 81    034 81    054 43    074 43
015 46    035 40    055 00    075 00
016 12    036 65    056 03    076 03
017 42    037 43    057 40    077 40
018 00    038 00    058 40    078 44
019 04    039 03    059 44    079 01
```

# APPENDIX A

## TEXAS INSTRUMENTS PRERECORDED CARDS

# SIMULTANEOUS EQUATIONS IN 2 OR 3 UNKNOWNS

Given the coefficients of two simultaneous equations with two unknowns each, the solutions are effected as follows.

$$a_0x + a_1y = a_2$$
$$b_0x + b_1y = b_2$$

$$x = \frac{\begin{vmatrix} a_2 & a_1 \\ b_2 & b_1 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix}} = \frac{a_2b_1 - a_1b_2}{a_0b_1 - a_1b_0}$$

$$y = \frac{\begin{vmatrix} a_0 & a_2 \\ b_0 & b_2 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix}} = \frac{a_0b_2 - a_2b_0}{a_0b_1 - a_1b_0}, \text{ where } a_0b_1 - a_1b_0 \neq 0.$$

For three unknowns:

$$a_0x + a_1y + a_2z = a_3$$
$$b_0x + b_1y + b_2z = b_3$$
$$c_0x + c_1y + c_2z = c_3$$

The program solves for x in the first equation and substitutes the result into the second and third equations. Now only two, two-unknown equations exist and are solved as above.

NOTE: $a_0$, $b_0$, and $c_0 \neq 0$.

Example: Solve for x and y in the following two equations:

$5.2x + 3y = 2.56$

$10.9x - 4.3y = -34.87$

| Enter | Press | Display | Comments |
|---|---|---|---|
| | 2nd CMs | 0. | Clear data memories |
| **5.2** | 2nd A' | 5.2 | $a_0$ |
| **3** | RUN | 3. | $a_1$ |
| **2.56** | RUN | 2.56 | $a_2$ |
| **10.9** | 2nd B' | 10.9 | $b_0$ |
| **4.3** | +/− RUN | −4.3 | $b_1$ |
| **34.87** | +/− RUN | −34.87 | $b_2$ |
| | D | −1.7 | x |
| | RUN | 3.8 | y |

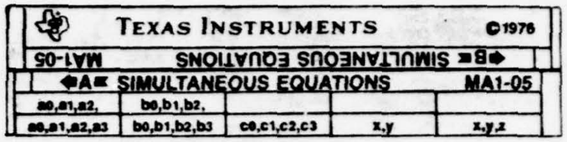
Example: Solve for x, y, and z.

$.25x - 3.1y - .3z = .675$

$1.25x + .18y + .73z = .901$

$-.8x + 1.1y - 1.6z = -2.074$

$x = .18, y = -.3, z = 1.0$



TEXAS INSTRUMENTS ©1976

◆B≡ SIMULTANEOUS EQUATIONS MA1-05

◆A≡ SIMULTANEOUS EQUATIONS MA1-05

| | | | | |
|---|---|---|---|---|
| a0,a1,a2, | b0,b1,b2, | | | |
| a0,a1,a2,a3 | b0,b1,b2,b3 | c0,c1,c2,c3 | x,y | x,y,z |

## USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | DISPLAY |
|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | |
| 2 | Clear Data Memories | | 2nd | CMs | |
| | Perform Steps 3-4 for | | | | |
| | 2 unknowns OR | | | | |
| | Perform Steps 5-6 for | | | | |
| | 3 unknowns | | | | |
| 3* | Enter coefficients for | $a_0$ | 2nd | A' | $a_0$ |
| | 2 unknowns | $a_1$ | RUN | | $a_1$ |
| | | $a_2$ | RUN | | $a_2$ |
| | | $b_1$ | 2nd | B' | $b_1$ |
| | | $b_2$ | RUN | | $b_2$ |
| | | $b_3$ | RUN | | $b_3$ |
| 4 | Calculate x and y | | D | | x |
| | | | RUN | | y |
| 5* | Enter coefficients for | $a_0$ | A | | $a_0$ |
| | 3 unknowns | $a_1$ | RUN | | $a_1$ |
| | | $a_2$ | RUN | | $a_2$ |
| | | $a_3$ | RUN | | $a_3$ |
| | | $b_0$ | B | | $b_0$ |
| | | $b_1$ | RUN | | $b_1$ |
| | | $b_2$ | RUN | | $b_2$ |
| | | $b_3$ | RUN | | $b_3$ |
| | | $c_0$ | C | | $c_0$ |
| | | $c_1$ | RUN | | $c_1$ |
| | | $c_2$ | RUN | | $c_2$ |
| | | $c_3$ | RUN | | $c_3$ |
| 6 | Calculate x, y, and z | | E | | x |
| | | | RUN | | y |
| | | | RUN | | z |

*NOTE: To correct input error return to last user defined key and reenter associated values

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 06 | 6 | 056 03 | 3 | 084 65 | × |
| 001 11 | A | 029 56 | *rtn | 057 56 | *rtn | 085 43 | RCL |
| 002 42 | STO | 030 42 | STO | 058 46 | *LBL | 086 01 | 1 |
| 003 00 | 0 | 031 00 | 0 | 059 14 | D | 087 01 | 1 |
| 004 00 | 0 | 032 07 | 7 | 060 10 | *E' | 088 95 | = |
| 005 56 | *rtn | 033 56 | *rtn | 061 46 | *LBL | 089 55 | ÷ |
| 006 42 | STO | 034 42 | STO | 062 85 | + | 090 53 | ( |
| 007 00 | 0 | 035 00 | 0 | 063 43 | RCL | 091 43 | RCL |
| 008 01 | 1 | 036 08 | 8 | 064 00 | 0 | 092 00 | 0 |
| 009 56 | *rtn | 037 56 | *rtn | 065 09 | 9 | 093 06 | 6 |
| 010 42 | STO | 038 46 | *LBL | 066 56 | *rtn | 094 65 | × |
| 011 00 | 0 | 039 13 | C | 067 43 | RCL | 095 43 | RCL |
| 012 02 | 2 | 040 42 | STO | 068 00 | 0 | 096 01 | 1 |
| 013 56 | *rtn | 041 01 | 1 | 069 04 | 4 | 097 02 | 2 |
| 014 42 | STO | 042 00 | 0 | 070 56 | *rtn | 098 75 | − |
| 015 00 | 0 | 043 56 | *rtn | 071 46 | *LBL | 099 43 | RCL |
| 016 03 | 3 | 044 46 | *LBL | 072 10 | *E' | 100 00 | 0 |
| 017 56 | *rtn | 045 17 | *B' | 073 43 | RCL | 101 07 | 7 |
| 018 46 | *LBL | 046 42 | STO | 074 00 | 0 | 102 65 | × |
| 019 12 | B | 047 01 | 1 | 075 06 | 6 | 103 43 | RCL |
| 020 42 | STO | 048 01 | 1 | 076 65 | × | 104 01 | 1 |
| 021 00 | 0 | 049 56 | *rtn | 077 43 | RCL | 105 01 | 1 |
| 022 05 | 5 | 050 42 | STO | 078 01 | 1 | 106 95 | = |
| 023 56 | *rtn | 051 01 | 1 | 079 03 | 3 | 107 42 | STO |
| 024 46 | *LBL | 052 02 | 2 | 080 75 | − | 108 00 | 0 |
| 025 16 | *A' | 053 56 | *rtn | 081 43 | RCL | 109 04 | 4 |
| 026 42 | STO | 054 42 | STO | 082 00 | 0 | 110 65 | × |
| 027 00 | 0 | 055 01 | 1 | 083 08 | 8 | 111 43 | RCL |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 $a_0$ | 05 $-b_0$ | 10 $-c_0$ | 15 |
| 01 $a_1$ | 06 $b_1, a_0$ | 11 $c_1, b_0$ | 16 |
| 02 $a_2$ | 07 $b_2, a_1$ | 12 $c_2, b_1$ | 17 |
| 03 $a_3$ | 08 $b_3, a_2$ | 13 $c_3, b_2$ | 18 |
| 04 z,y | 09 y,x | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 00 | 0 | 140 02 | 2 | 168 42 | STO | 196 95 | = |
| 113 07 | 7 | 141 49 | *PROD | 169 00 | 0 | 197 56 | *rtn |
| 114 94 | +/− | 142 00 | 0 | 170 09 | 9 | 198 41 | GTO |
| 115 85 | + | 143 03 | 3 | 171 02 | 2 | 199 85 | + |
| 116 43 | RCL | 144 43 | RCL | 172 44 | SUM | 200 46 | *LBL |
| 117 00 | 0 | 145 00 | 0 | 173 01 | 1 | 201 19 | *D' |
| 118 08 | 8 | 146 05 | 5 | 174 04 | 4 | 202 43 | RCL |
| 119 95 | = | 147 42 | STO | 175 19 | *D' | 203 00 | 0 |
| 120 55 | ÷ | 148 00 | 0 | 176 19 | *D' | 204 04 | 4 |
| 121 43 | RCL | 149 04 | 4 | 177 19 | *D' | 205 65 | × |
| 122 00 | 0 | 150 01 | 1 | 178 10 | *E' | 206 36 | *IND |
| 123 06 | 6 | 151 42 | STO | 179 65 | × | 207 43 | RCL |
| 124 95 | = | 152 00 | 0 | 180 43 | RCL | 208 00 | 0 |
| 125 42 | STO | 153 09 | 9 | 181 00 | 0 | 209 09 | 9 |
| 126 00 | 0 | 154 06 | 6 | 182 01 | 1 | 210 95 | = |
| 127 09 | 9 | 155 42 | STO | 183 94 | +/− | 211 94 | +/− |
| 128 56 | *rtn | 156 01 | 1 | 184 75 | − | 212 36 | *IND |
| 129 46 | *LBL | 157 04 | 4 | 185 43 | RCL | 213 44 | SUM |
| 130 15 | E | 158 19 | *D' | 186 00 | 0 | 214 01 | 1 |
| 131 43 | RCL | 159 19 | *D' | 187 04 | 4 | 215 04 | 4 |
| 132 00 | 0 | 160 19 | *D' | 188 65 | × | 216 01 | 1 |
| 133 00 | 0 | 161 43 | RCL | 189 43 | RCL | 217 44 | SUM |
| 134 20 | *1/x | 162 01 | 1 | 190 00 | 0 | 218 00 | 0 |
| 135 49 | *PROD | 163 00 | 0 | 191 02 | 2 | 219 09 | 9 |
| 136 00 | 0 | 164 42 | STO | 192 85 | + | 220 44 | SUM |
| 137 01 | 1 | 165 00 | 0 | 193 43 | RCL | 221 01 | 1 |
| 138 49 | *PROD | 166 04 | 4 | 194 00 | 0 | 222 04 | 4 |
| 139 00 | 0 | 167 01 | 1 | 195 03 | 3 | 223 56 | *rtn |

*Denotes 2nd function key

## MATRIX INVERSION AND DETERMINANT (3 x 3)

This program will evaluate and take the inverse of a 3 x 3 matrix.

$$\text{Matrix } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

The determinant of matrix A is:

$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

The inverse of matrix A is $B = A^{-1}$, where $A \times A^{-1} = 1$.

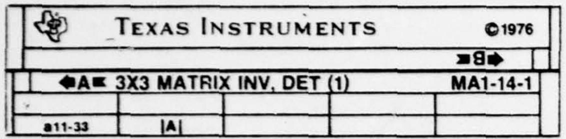
The elements of inverted matrix B are:

$$b_{11} = (a_{22}a_{33} - a_{32}a_{23}) \;/\; |A|$$
$$b_{21} = (a_{32}a_{13} - a_{12}a_{33}) \;/\; |A|$$
$$b_{31} = (a_{12}a_{23} - a_{22}a_{13}) \;/\; |A|$$
$$b_{12} = (a_{31}a_{23} - a_{21}a_{33}) \;/\; |A|$$
$$b_{22} = (a_{11}a_{33} - a_{31}a_{13}) \;/\; |A|$$
$$b_{32} = (a_{21}a_{13} - a_{11}a_{23}) \;/\; |A|$$
$$b_{13} = (a_{21}a_{32} - a_{31}a_{22}) \;/\; |A|$$
$$b_{23} = (a_{31}a_{12} - a_{11}a_{32}) \;/\; |A|$$
$$b_{33} = (a_{11}a_{22} - a_{21}a_{12}) \;/\; |A|$$

NOTES: 1. If $|A| = 0$, the display will flash 0 when $\boxed{C}$ is pressed.
2. The determinant must be calculated before its inverse.

Example: Let $A = \begin{pmatrix} 2 & 4 & 9 \\ 1 & 3 & 5 \\ 7 & 8 & 6 \end{pmatrix}$. Evaluate $|A|$ and find $A^{-1}$

| Enter | Press | Display | Comments |
|---|---|---|---|
| 2 | $\boxed{A}$ | 2. | $a_{11}$ |
| 4 | $\boxed{RUN}$ | 4. | $a_{12}$ |
| 9 | $\boxed{RUN}$ | 9. | $a_{13}$ |
| 1 | $\boxed{RUN}$ | 1. | $a_{21}$ |
| 3 | $\boxed{RUN}$ | 3. | $a_{22}$ |
| 5 | $\boxed{RUN}$ | 5. | $a_{23}$ |
| 7 | $\boxed{RUN}$ | 7. | $a_{31}$ |
| 8 | $\boxed{RUN}$ | 8. | $a_{32}$ |
| 6 | $\boxed{RUN}$ | 6. | $a_{33}$ |
|  | $\boxed{B}$ | −45. | $\lvert A\rvert$ |
| Enter card 2 |  |  |  |
|  | $\boxed{C}$ | .4888888889 | $b_{11}$ |
|  | $\boxed{RUN}$ | −1.066666667 | $b_{21}$ |
|  | $\boxed{RUN}$ | .1555555556 | $b_{31}$ |
|  | $\boxed{RUN}$ | −.6444444444 | $b_{12}$ |
|  | $\boxed{RUN}$ | 1.133333333 | $b_{22}$ |
|  | $\boxed{RUN}$ | .0222222222 | $b_{32}$ |
|  | $\boxed{RUN}$ | .2888888889 | $b_{13}$ |
|  | $\boxed{RUN}$ | −.2666666667 | $b_{23}$ |
|  | $\boxed{RUN}$ | −.0444444444 | $b_{33}$ |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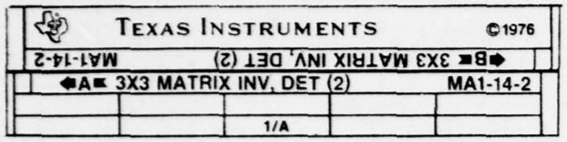
B

A 3X3 MATRIX INV, DET (1) MA1-14-1

a11-33 |A|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B 3X3 MATRIX INV, DET (2) MA1-14-2

A 3X3 MATRIX INV, DET (2) MA1-14-2

1/A

## USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Card 1 (Side A) | | | | | |
| 2 | Enter Matrix A | $a_{11}$ | A | | | $a_{11}$ |
| | | $a_{12}$ | RUN | | | $a_{12}$ |
| | | $a_{13}$ | RUN | | | $a_{13}$ |
| | | $a_{21}$ | RUN | | | $a_{21}$ |
| | | $a_{22}$ | RUN | | | $a_{22}$ |
| | | $a_{23}$ | RUN | | | $a_{23}$ |
| | | $a_{31}$ | RUN | | | $a_{31}$ |
| | | $a_{32}$ | RUN | | | $a_{32}$ |
| | | $a_{33}$ | RUN | | | $a_{33}$ |
| 3 | Calculate determinant | | B | | | \|A\| |
| 4 | Enter Card 2 (A and B) | | | | | |
| 5 | Calculate inverse matrix | | C | | | $b_{11}$ |
| | | | RUN | | | $b_{21}$ |
| | | | RUN | | | $b_{31}$ |
| | | | RUN | | | $b_{12}$ |
| | | | RUN | | | $b_{22}$ |
| | | | RUN | | | $b_{32}$ |
| | | | RUN | | | $b_{13}$ |
| | | | RUN | | | $b_{23}$ |
| | | | RUN | | | $b_{33}$ |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 12 | B | 056 03 | 3 | 084 08 | 8 |
| 001 11 | A | 029 43 | RCL | 057 65 | × | 085 65 | × |
| 002 48 | *EXC | 030 00 | 0 | 058 43 | RCL | 086 43 | RCL |
| 003 00 | 0 | 031 09 | 9 | 059 00 | 0 | 087 00 | 0 |
| 004 00 | 0 | 032 65 | × | 060 04 | 4 | 088 06 | 6 |
| 005 01 | 1 | 033 43 | RCL | 061 65 | × | 089 75 | − |
| 006 48 | *EXC | 034 00 | 0 | 062 43 | RCL | 090 43 | RCL |
| 007 00 | 0 | 035 01 | 1 | 063 00 | 0 | 091 00 | 0 |
| 008 00 | 0 | 036 65 | × | 064 08 | 8 | 092 04 | 4 |
| 009 46 | *LBL | 037 43 | RCL | 065 75 | − | 093 65 | × |
| 010 68 | *8' | 038 00 | 0 | 066 43 | RCL | 094 43 | RCL |
| 011 36 | *IND | 039 05 | 5 | 067 00 | 0 | 095 00 | 0 |
| 012 42 | STO | 040 85 | + | 068 07 | 7 | 096 02 | 2 |
| 013 00 | 0 | 041 53 | ( | 069 65 | × | 097 65 | × |
| 014 00 | 0 | 042 43 | RCL | 070 43 | RCL | 098 43 | RCL |
| 015 56 | *rtn | 043 00 | 0 | 071 00 | 0 | 099 00 | 0 |
| 016 48 | *EXC | 044 02 | 2 | 072 05 | 5 | 100 09 | 9 |
| 017 00 | 0 | 045 65 | × | 073 65 | × | 101 95 | = |
| 018 00 | 0 | 046 43 | RCL | 074 43 | RCL | 102 42 | STO |
| 019 85 | + | 047 00 | 0 | 075 00 | 0 | 103 01 | 1 |
| 020 01 | 1 | 048 06 | 6 | 076 03 | 3 | 104 09 | 9 |
| 021 95 | = | 049 65 | × | 077 75 | − | 105 56 | *rtn |
| 022 48 | *EXC | 050 43 | RCL | 078 43 | RCL | 106 00 | |
| 023 00 | 0 | 051 00 | 0 | 079 00 | 0 | 107 00 | |
| 024 00 | 0 | 052 07 | 7 | 080 01 | 1 | 108 00 | |
| 025 41 | GTO | 053 85 | + | 081 65 | × | 109 00 | |
| 026 68 | *8' | 054 43 | RCL | 082 43 | RCL | 110 00 | |
| 027 46 | *LBL | 055 00 | 0 | 083 00 | 0 | 111 00 | |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 | 05 $a_{22}$ | 10 | 15 |
| 01 $a_{11}$ | 06 $a_{23}$ | 11 | 16 |
| 02 $a_{12}$ | 07 $a_{31}$ | 12 | 17 |
| 03 $a_{13}$ | 08 $a_{32}$ | 13 | 18 |
| 04 $a_{21}$ | 09 $a_{33}$ | 14 | 19 Det. |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 75 | – | 056 42 | STO | 084 02 | 2 |
| 001 13 | C | 029 43 | RCL | 057 01 | 1 | 085 75 | – |
| 002 43 | RCL | 030 00 | 0 | 058 01 | 1 | 086 43 | RCL |
| 003 00 | 0 | 031 07 | 7 | 059 43 | RCL | 087 00 | 0 |
| 004 07 | 7 | 032 65 | × | 060 00 | 0 | 088 01 | 1 |
| 005 65 | × | 033 43 | RCL | 061 01 | 1 | 089 65 | × |
| 006 43 | RCL | 034 00 | 0 | 062 65 | × | 090 43 | RCL |
| 007 00 | 0 | 035 05 | 5 | 063 43 | RCL | 091 00 | 0 |
| 008 06 | 6 | 036 10 | *E' | 064 00 | 0 | 092 08 | 8 |
| 009 75 | – | 037 43 | STO | 065 09 | 9 | 093 10 | *E' |
| 010 43 | RCL | 038 01 | 1 | 066 75 | – | 094 42 | STO |
| 011 00 | 0 | 039 06 | 6 | 067 43 | RCL | 095 01 | 1 |
| 012 04 | 4 | 040 43 | RCL | 068 00 | 0 | 096 07 | 7 |
| 013 65 | × | 041 00 | 0 | 069 07 | 7 | 097 43 | RCL |
| 014 43 | RCL | 042 08 | 8 | 070 65 | × | 098 00 | 0 |
| 015 00 | 0 | 043 65 | × | 071 43 | RCL | 099 02 | 2 |
| 016 09 | 9 | 044 43 | RCL | 072 00 | 0 | 100 65 | × |
| 017 10 | *E' | 045 00 | 0 | 073 03 | 3 | 101 43 | RCL |
| 018 42 | STO | 046 03 | 3 | 074 10 | *E' | 102 00 | 0 |
| 019 01 | 1 | 047 75 | – | 075 42 | STO | 103 06 | 6 |
| 020 03 | 3 | 048 43 | RCL | 076 01 | 1 | 104 75 | – |
| 021 43 | RCL | 049 00 | 0 | 077 04 | 4 | 105 43 | RCL |
| 022 00 | 0 | 050 02 | 2 | 078 43 | RCL | 106 00 | 0 |
| 023 04 | 4 | 051 65 | × | 079 00 | 0 | 107 05 | 5 |
| 024 65 | × | 052 43 | RCL | 080 07 | 7 | 108 65 | × |
| 025 43 | RCL | 053 00 | 0 | 081 65 | × | 109 43 | RCL |
| 026 00 | 0 | 054 09 | 9 | 082 43 | RCL | 110 00 | 0 |
| 027 08 | 8 | 055 10 | *E' | 083 00 | 0 | 111 03 | 3 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 | 05 $a_{22}$ | 10 $b_{11}$ | 15 $b_{23}$ |
| 01 $a_{11}$ | 06 $a_{23}$ | 11 $b_{12}$ | 16 $b_{31}$ |
| 02 $a_{12}$ | 07 $a_{31}$ | 12 $b_{13}$ | 17 $b_{32}$ |
| 03 $a_{13}$ | 08 $a_{32}$ | 13 $b_{21}$ | 18 $b_{33}$ |
| 04 $a_{21}$ | 09 $a_{33}$ | 14 $b_{22}$ | 19 Det. |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 10 | *E' | 140 00 | 0 | 168 06 | 6 | 196 43 | RCL |
| 113 42 | STO | 141 05 | 5 | 169 10 | *E' | 197 01 | 1 |
| 114 01 | 1 | 142 75 | – | 170 42 | STO | 198 09 | 9 |
| 115 02 | 2 | 143 43 | RCL | 171 01 | 1 | 199 95 | = |
| 116 43 | RCL | 144 00 | 0 | 172 00 | 0 | 200 70 | *if err |
| 117 00 | 0 | 145 04 | 4 | 173 56 | *rtn | 201 67 | *7' |
| 118 04 | 4 | 146 65 | × | 174 01 | 1 | 202 56 | *rtn |
| 119 65 | × | 147 43 | RCL | 175 01 | 1 | 203 46 | *LBL |
| 120 43 | RCL | 148 00 | 0 | 176 42 | STO | 204 67 | *7' |
| 121 00 | 0 | 149 02 | 2 | 177 00 | 0 | 205 00 | 0 |
| 122 03 | 3 | 150 10 | *E' | 178 00 | 0 | 206 20 | *1/x |
| 123 75 | – | 151 42 | STO | 179 46 | *LBL | 207 00 | 0 |
| 124 43 | RCL | 152 01 | 1 | 180 89 | *3' | 208 56 | *rtn |
| 125 00 | 0 | 153 08 | 8 | 181 36 | *IND | 209 00 | |
| 126 01 | 1 | 154 43 | RCL | 182 43 | RCL | 210 00 | |
| 127 65 | × | 155 00 | 0 | 183 00 | 0 | 211 00 | |
| 128 43 | RCL | 156 05 | 5 | 184 00 | 0 | 212 00 | |
| 129 00 | 0 | 157 65 | × | 185 56 | *rtn | 213 00 | |
| 130 06 | 6 | 158 43 | RCL | 186 01 | 1 | 214 00 | |
| 131 10 | *E' | 159 00 | 0 | 187 44 | SUM | 215 00 | |
| 132 42 | STO | 160 09 | 9 | 188 00 | 0 | 216 00 | |
| 133 01 | 1 | 161 75 | – | 189 00 | 0 | 217 00 | |
| 134 05 | 5 | 162 43 | RCL | 190 41 | GTO | 218 00 | |
| 135 43 | RCL | 163 00 | 0 | 191 89 | *3' | 219 00 | |
| 136 00 | 0 | 164 08 | 8 | 192 46 | *LBL | 220 00 | |
| 137 01 | 1 | 165 65 | × | 193 10 | *E' | 221 00 | |
| 138 65 | × | 166 43 | RCL | 194 95 | = | 222 00 | |
| 139 43 | RCL | 167 00 | 0 | 195 55 | ÷ | 223 00 | |

*Denotes 2nd function key

## MATRIX ARITHMETIC (2)

The product AB in that order of the m x p matrix A = $[a_{ij}]$ and the p x n matrix B = $[b_{ij}]$ is the m x n matrix C = $[c_{ij}]$ where $0 \le m \le 3$, $0 \le n \le 3, 0 \le p \le 3$.

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = C$$

NOTE: if $m < 3$, $n < 3$, $p < 3$, it is not necessary to enter zeros for the remainder of the matrix elements. Matrix 'A' is not destroyed, thus you may do constant multiplication by matrix 'A'.

Reference: *Computer Methods for Science and Engineering*, Robert L. LaFara, Hayden Book Company, 1973.

Example: Evaluate $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

| **Enter** | **Press** | **Display** | **Comments** |
|---|---|---|---|
| | E | 0. | Initialize |
| **2** | A | 1. | $a_{11}$ |
| **3** | RUN | 1. | $a_{12}$ |
| **1** | B | 1. | $a_{21}$ |
| **2** | RUN | 1. | $a_{22}$ |
| | D | 1. | Multiply |
| **1** | A | 0. | $a_{31} \times b_{11}$ |
| **2** | RUN | 0. | $a_{31} \times b_{21}$ |
| **3** | B | 0. | $a_{32} \times b_{12}$ |
| **4** | RUN | 0. | $a_{32} \times b_{22}$ |
| | 2nd A' | 11. | $c_{11}$ |
| | RUN | 16. | $c_{12}$ |
| | 2nd B' | 7. | $c_{21}$ |
| | RUN | 10. | $c_{22}$ |

Example: $[1\ 2\ 3] \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = [32]$

Example: $\begin{bmatrix} 1 & 4 & -2 \\ 6 & 3 & 0 \\ 6 & -1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ -1 & 2 & 6 \\ 4 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -12 & 6 & 22 \\ -3 & 30 & 30 \\ -7 & 16 & 2 \end{bmatrix}$

TEXAS INSTRUMENTS ©1976

MA1-16 ◆B≡ MATRIX ARITHMETIC (2)

◆A≡ MATRIX ARITHMETIC (2) MA1-16

| | | | | |
|---|---|---|---|---|
| c1j | c2j | c3j | | |
| a1j (b1j) | a2j (b2j) | a3j (b3j) | MULT | INIT |

### USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | *Enter Program (A and B)* | | | | | |
| 2 | Initialize | | E | | | |
| 3 | Enter matrix | $a_{11}$ $(b_{11})$ | A | | | $a_{31} \times b_{11}$ |
| 4 | | $a_{12}$ $(b_{12})$ | RUN | | | $a_{31} \times b_{12}$ |
| 5 | | $a_{13}$ $(b_{13})$ | RUN | | | $a_{31} \times b_{13}$ |
| 6 | | $a_{21}$ $(b_{21})$ | B | | | $a_{32} \times b_{21}$ |
| 7 | | $a_{22}$ $(b_{22})$ | RUN | | | $a_{32} \times b_{22}$ |
| 8 | | $a_{23}$ $(b_{23})$ | RUN | | | $a_{32} \times b_{23}$ |
| 9 | | $a_{31}$ $(b_{31})$ | C | | | $a_{33} \times b_{31}$ |
| 10 | | $a_{32}$ $(b_{32})$ | RUN | | | $a_{33} \times b_{32}$ |
| 11 | | $a_{33}$ $(b_{33})$ | RUN | | | $a_{33} \times b_{33}$ |
| | (If Matrix B has been | | | | | |
| | entered go to 13) | | | | | |
| 12 | Setup for multiply | | D | | | |
| | (Repeat 3-11 for Matrix B) | | | | | |
| 13 | Recall Matrix C | | 2nd | A' | | $c_{11}$ |
| | | | RUN | | | $c_{12}$ |
| | | | RUN | | | $c_{13}$ |
| | | | 2nd | B' | | $c_{21}$ |
| | | | RUN | | | $c_{22}$ |
| | | | RUN | | | $c_{23}$ |
| | | | 2nd | C' | | $c_{31}$ |
| | | | RUN | | | $c_{32}$ |
| | | | RUN | | | $c_{33}$ |
| 14 | To use Matrix A again. —— | go to Step 12 | | | | |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 25 | CLR | 028 52 | EE | 056 56 | *rtn | 084 00 | 0 |
| 001 56 | *rtn | 029 19 | *D' | 057 46 | *LBL | 085 00 | 0 |
| 002 46 | *LBL | 030 44 | SUM | 058 42 | STO | 086 07 | 7 |
| 003 15 | E | 031 00 | 0 | 059 19 | *D' | 087 51 | SBR |
| 004 47 | *CMs | 032 01 | 1 | 060 44 | SUM | 088 24 | CE |
| 005 86 | *rset | 033 10 | *E' | 061 00 | 0 | 089 56 | *rtn |
| 006 46 | *LBL | 034 44 | SUM | 062 03 | 3 | 090 42 | STO |
| 007 19 | *D' | 035 00 | 0 | 063 10 | *E' | 091 00 | 0 |
| 008 42 | STO | 036 04 | 4 | 064 44 | SUM | 092 00 | 0 |
| 009 01 | 1 | 037 10 | *E' | 065 00 | 0 | 093 07 | 7 |
| 010 09 | 9 | 038 44 | SUM | 066 06 | 6 | 094 51 | SBR |
| 011 46 | *LBL | 039 00 | 0 | 067 10 | *E' | 095 42 | STO |
| 012 10 | *E' | 040 07 | 7 | 068 44 | SUM | 096 56 | *rtn |
| 013 03 | 3 | 041 56 | *rtn | 069 00 | 0 | 097 46 | *LBL |
| 014 44 | SUM | 042 46 | *LBL | 070 09 | 9 | 098 88 | *2' |
| 015 01 | 1 | 043 24 | CE | 071 56 | *rtn | 099 08 | 8 |
| 016 09 | 9 | 044 19 | *D' | 072 46 | *LBL | 100 51 | SBR |
| 017 43 | RCL | 045 44 | SUM | 073 14 | D | 101 52 | EE |
| 018 00 | 0 | 046 00 | 0 | 074 50 | *st flg | 102 56 | *rtn |
| 019 00 | 0 | 047 02 | 2 | 075 00 | 0 | 103 42 | STO |
| 020 65 | × | 048 10 | *E' | 076 56 | *rtn | 104 00 | 0 |
| 021 36 | *IND | 049 44 | SUM | 077 46 | *LBL | 105 00 | 0 |
| 022 43 | RCL | 050 00 | 0 | 078 87 | *1' | 106 08 | 8 |
| 023 01 | 1 | 051 05 | 5 | 079 07 | 7 | 107 51 | SBR |
| 024 09 | 9 | 052 10 | *E' | 080 51 | SBR | 108 24 | CE |
| 025 95 | = | 053 44 | SUM | 081 52 | EE | 109 56 | *rtn |
| 026 56 | *rtn | 054 00 | 0 | 082 56 | *rtn | 110 42 | STO |
| 027 46 | *LBL | 055 08 | 8 | 083 42 | STO | 111 00 | 0 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 Used | 05 $c_{22}$ | 10 $a_{11}$ | 15 $a_{23}$ |
| 01 $c_{11}$ | 06 $c_{23}$ | 11 $a_{12}$ | 16 $a_{31}$ |
| 02 $c_{12}$ | 07 $c_{31}$ | 12 $a_{13}$ | 17 $a_{32}$ |
| 03 $c_{13}$ | 08 $c_{32}$ | 13 $a_{21}$ | 18 $a_{33}$ |
| 04 $c_{21}$ | 09 $c_{33}$ | 14 $a_{22}$ | 19 Used |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 Used | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 00 | 0 | 140 00 | 0 | 168 00 | 0 | 196 00 | 0 |
| 113 08 | 8 | 141 00 | 0 | 169 00 | 0 | 197 46 | *LBL |
| 114 51 | SBR | 142 60 | *if flg | 170 60 | *if flg | 198 65 | × |
| 115 42 | STO | 143 00 | 0 | 171 00 | 0 | 199 01 | 1 |
| 116 56 | *rtn | 144 87 | *1' | 172 88 | *2' | 200 44 | SUM |
| 117 46 | *LBL | 145 01 | 1 | 173 01 | 1 | 201 00 | 0 |
| 118 89 | *3' | 146 00 | 0 | 174 03 | 3 | 202 00 | 0 |
| 119 09 | 9 | 147 46 | *LBL | 175 41 | GTO | 203 25 | CLR |
| 120 51 | SBR | 148 32 | sin | 176 32 | sin | 204 36 | *IND |
| 121 52 | EE | 149 48 | *EXC | 177 46 | *LBL | 205 43 | RCL |
| 122 56 | *rtn | 150 00 | 0 | 178 13 | C | 206 00 | 0 |
| 123 42 | STO | 151 00 | 0 | 179 42 | STO | 207 00 | 0 |
| 124 00 | 0 | 152 46 | *LBL | 180 00 | 0 | 208 56 | *rtn |
| 125 00 | 0 | 153 33 | cos | 181 00 | 0 | 209 41 | GTO |
| 126 09 | 9 | 154 36 | *IND | 182 60 | *if flg | 210 65 | × |
| 127 51 | SBR | 155 42 | STO | 183 00 | 0 | 211 46 | *LBL |
| 128 24 | CE | 156 00 | 0 | 184 89 | *3' | 212 17 | *B' |
| 129 56 | *rtn | 157 00 | 0 | 185 01 | 1 | 213 03 | 3 |
| 130 42 | STO | 158 01 | 1 | 186 06 | 6 | 214 41 | GTO |
| 131 00 | 0 | 159 44 | SUM | 167 41 | GTO | 215 85 | + |
| 132 00 | 0 | 160 00 | 0 | 188 32 | sin | 216 46 | *LBL |
| 133 09 | 9 | 161 00 | 0 | 189 46 | *LBL | 217 18 | *C' |
| 134 51 | SBR | 162 56 | *rtn | 190 16 | *A' | 218 06 | 6 |
| 135 42 | STO | 163 41 | GTO | 191 00 | 0 | 219 41 | GTO |
| 136 56 | *rtn | 164 33 | cos | 192 46 | *LBL | 220 85 | + |
| 137 46 | *LBL | 165 46 | *LBL | 193 85 | + | 221 00 | |
| 138 11 | A | 166 12 | B | 194 42 | STO | 222 00 | |
| 139 42 | STO | 167 42 | STO | 195 00 | 0 | 223 00 | |

*Denotes 2nd function key

# BASIC STATISTICS FOR ONE OR TWO VARIABLES

This program calculates means, standard deviations, and standard errors of the mean for one or two variables, and covariance and correlation coefficient for two variables.

The input is a set of data points:
$\{(x_1,y_1),(x_2,y_2),\ldots(x_i,y_i),\ldots(x_n,y_n)\}$

in which the $y_i$'s may be omitted if only one variable is being used.

The statistics computed are:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \text{mean of } x$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \text{mean of } y$$

$$S_x = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}} = \text{std. dev. of x by n}-1 \text{ method}$$

$$S'_x = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n}} = \text{std. dev. of x by n method}$$

$$S_y = \sqrt{\frac{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}{n-1}} = \text{std. dev. of y by n}-1 \text{ method}$$

$$S'_y = \sqrt{\frac{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}{n}} = \text{std. dev. of y by n method}$$

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}} = \text{std. error of the mean of x by n}-1 \text{ method}$$

$$S'_{\bar{x}} = \frac{S'_x}{\sqrt{n}} = \text{std. error of the mean of x by n method}$$

$$S_{\bar{y}} = \frac{S_y}{\sqrt{n}} = \text{std. error of the mean of y by n}-1 \text{ method}$$

$$S'_{\bar{y}} = \frac{S'_y}{\sqrt{n}} = \text{std. error of the mean of y by n method}$$

$$S_{xy} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right] = \text{covariance of x and y by n}-1 \text{ method}$$

$$S'_{xy} = \frac{1}{n}\left[\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right] = \text{covariance of x and y by n method}$$

$$r_{xy} = \frac{S_{xy}}{S_x S_y} = \frac{S'_{xy}}{S'_x S'_y} = \text{correlation coefficient of x and y}$$

NOTES: 1. The number of data points must be > 1.
2. This program may be used with the Linear Regression program (ST1-08) to determine additional statistics. Data points entered on either program may be used for the other. Do not initialize if data from the other program is being used.

Example: Calculate $\bar{x}$, $\bar{y}$, $S_x$, $S'_x$, $S_y$, $S'_y$, $S_{\bar{x}}$, $S'_{\bar{x}}$, $S_{\bar{y}}$, $S'_{\bar{y}}$, $S_{xy}$, $S'_{xy}$, $r_{xy}$ for the data in the following table, then change the x and y values for i = 5 from (23, 17) to (5, 19) and find the new $r_{xy}$ and $S'_{xy}$.

| $i$ | $x_i$ | $y_i$ |
|---|---|---|
| 1 | 31 | 6 |
| 2 | 20.5 | 19 |
| 3 | 14 | 14 |
| 4 | 9 | 12.5 |
| 5 | 23 | 17 |
| 6 | 37 | 22 |
| 7 | 38 | 19.5 |
| 8 | 18 | 21 |
| 9 | 29 | 16 |

| Enter | Press | Display | Comment |
|---|---|---|---|
|  | 2nd CMs |  | Initialize |
| **31** | A | 1. | $x_1$ |
| **6** | RUN | 1. | $y_1$ |
| **20.5** | A | 2. | $x_2$ |
| **19** | RUN | 2. | $y_2$ |
| **14** | A | 3. | $x_3$ |
| **14** | RUN | 3. | $y_3$ |
| **etc.** |  |  |  |
|  | B | 24.38888889 | compute $\bar{x}$ |
|  | 2nd B' | 16.33333333 | compute $\bar{y}$ |
|  | C | 10.07403152 | compute $S_x$ |
|  | RUN | 9.497888004 | compute $S'_x$ |
|  | 2nd C' | 4.981214711 | compute $S_y$ |
|  | RUN | 4.696334268 | compute $S'_y$ |
|  | D | 3.358010507 | compute $S_{\bar{x}}$ |
|  | RUN | 3.165962668 | compute $S'_{\bar{x}}$ |
|  | 2nd D' | 1.660404904 | compute $S_{\bar{y}}$ |
|  | RUN | 1.565444756 | compute $S'_{\bar{y}}$ |
|  | E | 10.85416667 | compute $S_{xy}$ |
|  | RUN | 9.648148148 | compute $S'_{xy}$ |
|  | 2nd E' | .2163006968 | compute $r_{xy}$ |
| **23** | 2nd A' | 8. | delete 23 (23, 17) |
| **17** | RUN | 8. | delete 17 (23, 17) |
| **5** | A | 9. | insert 5 (5, 19) |
| **19** | RUN | 9. | insert 19 (5, 19) |
|  | 2nd E' | 0825567005 | compute new $r_{xy}$ |
|  | E | 5.006944445 | compute new $S_{xy}$ |
|  | RUN | 4.450617284 | compute new $S'_{xy}$ |

TEXAS INSTRUMENTS

◆B■ STATS FOR 1 OR 2 VARIABLES ST1-01

| ◆A■ STATS FOR 1 OR 2 VARIABLES |  |  |  | ST1-01 |
|---|---|---|---|---|
| **delete** | **$\bar{y}$, n** | **Sy, Sy'** | **$S\bar{y}$, $S\bar{y}'$** | **rxy** |
| **x, y** | **$\bar{x}$, n** | **Sx, Sx'** | **$S\bar{x}$, $S\bar{x}'$** | **Sxy, Sxy'** |

## USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS |  | DISPLAY |
|---|---|---|---|---|---|
| 1 | Enter Program (A and B) |  |  |  |  |
| 2 | Initialize |  | 2nd | CMs |  |
| 3 | Enter Data | $x_1$ | A |  | 1 |
|  | If only one | $y_1$ | RUN |  | 1 |
|  | variable is used | $x_2$ | A |  | 2 |
|  | omit the y steps | $y_2$ | RUN |  | 2 |
| 4 | To delete ($x_i$, $y_i$) | $x_i$ | 2nd | A' |  |
|  |  | $y_i$ | RUN |  |  |
| 5 | Compute outputs |  | B |  | $\bar{x}$ |
|  |  |  | RUN |  | n |
|  | All outputs except |  | 2nd | B' | $\bar{y}$ |
|  | those obtained by |  | RUN |  | n |
|  | pressing run, may |  | C |  | $S_x$ |
|  | be obtained in |  | RUN |  | $S'_x$ |
|  | any order. Outputs |  | 2nd | C' | $S_y$ |
|  | obtained by pressing |  | RUN |  | $S'_y$ |
|  | run must be |  | D |  | $S_{\bar{x}}$ |
|  | obtained immediately |  | RUN |  | $S'_{\bar{x}}$ |
|  | following the |  | 2nd | D' | $S_{\bar{y}}$ |
|  | corresponding |  | RUN |  | $S'_{\bar{y}}$ |
|  | keypush shown |  | E |  | $S_{xy}$ |
|  |  |  | RUN |  | $S'_{xy}$ |
|  |  |  | 2nd | E' | $r_{xy}$ |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 00 | 0 | 056 56 | *rtn | 084 44 | SUM |
| 001 11 | A | 029 56 | *rtn | 057 46 | *LBL | 085 00 | 0 |
| 002 42 | STO | 030 42 | STO | 058 16 | *A' | 086 04 | 4 |
| 003 00 | 0 | 031 00 | 0 | 059 95 | = | 087 40 | *$x^2$ |
| 004 01 | 1 | 032 02 | 2 | 060 94 | +/− | 088 94 | +/− |
| 005 44 | SUM | 033 44 | SUM | 061 42 | STO | 089 41 | GTO |
| 006 00 | 0 | 034 00 | 0 | 062 00 | 0 | 090 32 | sin |
| 007 03 | 3 | 035 04 | 4 | 063 01 | 1 | 091 46 | *LBL |
| 008 40 | *$x^2$ | 036 40 | *$x^2$ | 064 44 | SUM | 092 12 | B |
| 009 44 | SUM | 037 46 | *LBL | 065 00 | 0 | 093 43 | RCL |
| 010 00 | 0 | 038 32 | sin | 066 03 | 3 | 094 00 | 0 |
| 011 06 | 6 | 039 44 | SUM | 067 40 | *$x^2$ | 095 03 | 3 |
| 012 46 | *LBL | 040 00 | 0 | 068 94 | +/− | 096 46 | *LBL |
| 013 34 | tan | 041 07 | 7 | 069 44 | SUM | 097 33 | cos |
| 014 43 | RCL | 042 43 | RCL | 070 00 | 0 | 098 65 | X |
| 015 00 | 0 | 043 00 | 0 | 071 06 | 6 | 099 43 | RCL |
| 016 00 | 0 | 044 01 | 1 | 072 02 | 2 | 100 00 | 0 |
| 017 30 | *$\sqrt{x}$ | 045 65 | X | 073 94 | +/− | 101 00 | 0 |
| 018 42 | STO | 046 43 | RCL | 074 44 | SUM | 102 20 | *1/x |
| 019 00 | 0 | 047 00 | 0 | 075 00 | 0 | 103 54 | ) |
| 020 08 | 8 | 048 02 | 2 | 076 00 | 0 | 104 56 | *rtn |
| 021 01 | 1 | 049 95 | = | 077 51 | SBR | 105 43 | RCL |
| 022 44 | SUM | 050 44 | SUM | 078 34 | tan | 106 00 | 0 |
| 023 00 | 0 | 051 00 | 0 | 079 56 | *rtn | 107 00 | 0 |
| 024 00 | 0 | 052 05 | 5 | 080 42 | STO | 108 56 | *rtn |
| 025 25 | CLR | 053 43 | RCL | 081 00 | 0 | 109 46 | *LBL |
| 026 43 | RCL | 054 00 | 0 | 082 02 | 2 | 110 17 | *B' |
| 027 00 | 0 | 055 00 | 0 | 083 94 | +/− | 111 43 | RCL |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 n | 05 $\Sigma x_i y_i$ | 10 | 15 |
| 01 $x_i$ | 06 $\Sigma x_i^2$ | 11 | 16 |
| 02 $y_i$ | 07 $\Sigma y_i^2$ | 12 | 17 |
| 03 $\Sigma x_i$ | 08 $\sqrt{n-1}$ | 13 | 18 |
| 04 $\Sigma y_i$ | 09 | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 00 | 0 | 140 43 | RCL | 168 13 | C | 196 00 | 0 |
| 113 04 | 4 | 141 65 | X | 169 41 | GTO | 197 08 | 8 |
| 114 41 | GTO | 142 43 | RCL | 170 42 | STO | 198 40 | *$x^2$ |
| 115 33 | cos | 143 00 | 0 | 171 46 | *LBL | 199 95 | = |
| 116 46 | *LBL | 144 08 | 8 | 172 19 | *D' | 200 56 | *rtn |
| 117 13 | C | 145 46 | *LBL | 173 18 | *C' | 201 65 | X |
| 118 43 | RCL | 146 42 | STO | 174 41 | GTO | 202 43 | RCL |
| 119 00 | 0 | 147 55 | ÷ | 175 42 | STO | 203 00 | 0 |
| 120 06 | 6 | 148 43 | RCL | 176 46 | *LBL | 204 08 | 8 |
| 121 75 | − | 149 00 | 0 | 177 15 | E | 205 40 | *$x^2$ |
| 122 53 | ( | 150 00 | 0 | 178 43 | RCL | 206 55 | ÷ |
| 123 12 | B | 151 30 | *$\sqrt{x}$ | 179 00 | 0 | 207 43 | RCL |
| 124 46 | *LBL | 152 95 | = | 180 05 | 5 | 208 00 | 0 |
| 125 24 | CE | 153 56 | *rtn | 181 75 | − | 209 00 | 0 |
| 126 40 | *$x^2$ | 154 41 | GTO | 182 43 | RCL | 210 95 | = |
| 127 65 | X | 155 43 | RCL | 183 00 | 0 | 211 56 | *rtn |
| 128 43 | RCL | 156 46 | *LBL | 184 03 | 3 | 212 46 | *LBL |
| 129 00 | 0 | 157 18 | *C' | 185 65 | X | 213 10 | *E' |
| 130 00 | 0 | 158 43 | RCL | 186 43 | RCL | 214 15 | E |
| 131 54 | ) | 159 00 | 0 | 187 00 | 0 | 215 55 | ÷ |
| 132 30 | *$\sqrt{x}$ | 160 07 | 7 | 188 04 | 4 | 216 53 | ( |
| 133 55 | ÷ | 161 75 | − | 189 55 | ÷ | 217 53 | ( |
| 134 43 | RCL | 162 53 | ( | 190 43 | RCL | 218 13 | C |
| 135 00 | 0 | 163 17 | *B' | 191 00 | 0 | 219 55 | ÷ |
| 136 08 | 8 | 164 41 | GTO | 192 00 | 0 | 220 53 | ( |
| 137 95 | = | 165 24 | CE | 193 95 | = | 221 53 | ( |
| 138 56 | *rtn | 166 46 | *LBL | 194 55 | ÷ | 222 18 | *C' |
| 139 46 | *LBL | 167 14 | D | 195 43 | RCL | 223 56 | *rtn |

*Denotes 2nd function key

# MEANS AND MOMENTS

For a given set of input data, $\{x_1, x_2, \ldots x_n\}$, with associated frequencies, $\{f_1, f_2, \ldots f_n\}$ (for grouped data), the following means, moments, and skewness and kurtosis of distribution are calculated. If $f_i = 1$ for all i, the calculations are for ungrouped data, otherwise for grouped data.

**Means**

Arithmetic $= A = \frac{1}{N}\sum_{i=1}^{n} f_i x_i$

Harmonic $= H = N \Big/ \sum_{i=1}^{n} \frac{f_i}{x_i}$

Geometric $= G = \left(\prod_{i=1}^{n} x_i^{f_i}\right)^{1/N}$

Generalized $= M(t) = \left(\frac{1}{N}\sum_{i=1}^{n} f_i (x_i)^t\right)^{1/t}$

**Moments**

$m_1 = A = \frac{1}{N}\sum_{i=1}^{n} f_i x_i$

$m_2 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^2$

$m_3 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^3$

$m_4 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^4$

Kurtosis of distribution $= \frac{m_4}{(m_2)^2}$

Skewness of distribution $= \frac{m_3}{(m_2)^{3/2}}$

$N = \sum_{i=1}^{n} f_i$ $n$ = Number of different groups

NOTES:
1. The initialization routine assumes a frequency of 1 and $t = 1$.
2. The value of t may be changed before any $x_i$'s are entered, if changed after any $x_i$, M(t) will be meaningless.
3. For grouped data if $f_i = f_{i-1}$, it need not be entered.
4. A negative frequency will cause the next x entered to be deleted $|f|$ times.
5. A negative x will cause the geometric and generalized means to be invalid.

Error Indications:
A frequency or t of 0 will cause the display to flash 9.999999999 99 and the old value to be retained.

Example: The ungrouped ($f_i = 1$) data $\{x_i = 1.1, 2.3, 4.7, 3.5, 1.8, 2.9\}$ yields the following: A = 2.716666667, G = 2.451843924, H = 2.187595899, m(1) = 2.716666667; $m_2$ = 1.368055556, $m_3$ = .5367592591, $m_4$ = 3.903019215; kurtosis of distribution = 2.085418497, skewness of distribution = .3354470243

References: *Handbook of Mathematical Tables and Formulas*, R. S. Burington, McGraw-Hill, Fourth Edition, 1965, p. 164. *International Dictionary of Applied Mathematics*, W. F. Frieberger, D. Van Nostrand, 1960

TEXAS INSTRUMENTS

ST1-03-1 ◆B◼ MEANS & MOMENTS(1)

| ◆A◼ MEANS & MOMENTS(1) | | | | ST1-03-1 |
|---|---|---|---|---|
| A | G | H | M(t) | |
| x → Σf | f | t | del x | INIT |

TEXAS INSTRUMENTS

ST1-03-2 ◆B◼ MEANS & MOMENTS(2)

| ◆A◼ MEANS & MOMENTS(2) | | | | ST1-03-2 |
|---|---|---|---|---|
| A | G | H | M(t) | |
| m2 | m3 | m4 | kurt | skew |

## USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Program ST1-03-1 | | | | | |
| 2 | Initialize | | E | | | 1. |
| 3 | Enter t (for gen. mean) | $t \neq 0$ | C | | | t |
| 4 | Enter data | | | | | |
| | a. for grouped data, | | | | | |
| | ent. $f_i$ if $f_i \neq f_{i-1}$ | $f_i \neq 0$ | B | | | $f_i$ |
| | b. enter $x_i$ | $x_i$ | A | | | Σf |
| | repeat a and b for each i | | | | | |
| 5 | Delete data | | | | | |
| | a. for grouped data, | | | | | |
| | enter $f_i$ if $f_i \neq f_{i-1}$ | $f_i \neq 0$ | B | | | $f_i$ |
| | b. enter $x_i$ to be deleted | $x_i$ | D | | | corrected Σf |
| 6 | Calculate | | | | | |
| | a. arithmetic mean | | 2nd | A' | | A |
| | b. geometric mean | | 2nd | B' | | G |
| | c. harmonic mean | | 2nd | C' | | H |
| | d. generalized mean | | 2nd | D' | | M(t) |
| .7 | Enter Program ST1-03-2 | | | | | |
| 8 | Calculate | | | | | |
| | a. second moment | | A | | | $m_2$ |
| | b. third moment | | B | | | $m_3$ |
| | c. fourth moment | | C | | | $m_4$ |
| | d. kurtosis | | D | | | kurtosis |
| | e. skewness | | E | | | skewness |
| | f. means, see step 6 | | | | | |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 45 | $y^x$ | 056 43 | RCL | 084 00 | 0 |
| 001 11 | A | 029 43 | RCL | 057 00 | 0 | 085 05 | 5 |
| 002 42 | STO | 030 00 | 0 | 058 02 | 2 | 086 55 | ÷ |
| 003 00 | 0 | 031 02 | 2 | 059 65 | X | 087 43 | RCL |
| 004 04 | 4 | 032 95 | = | 060 44 | SUM | 088 00 | 0 |
| 005 65 | X | 033 49 | *PROD | 061 00 | 0 | 089 01 | 1 |
| 006 43 | RCL | 034 00 | 0 | 062 09 | 9 | 090 95 | = |
| 007 00 | 0 | 035 06 | 6 | 063 43 | RCL | 091 42 | STO |
| 008 02 | 2 | 036 43 | RCL | 064 00 | 0 | 092 01 | 1 |
| 009 95 | = | 037 00 | 0 | 065 04 | 4 | 093 02 | 2 |
| 010 44 | SUM | 038 04 | 4 | 066 65 | X | 094 25 | CLR |
| 011 00 | 0 | 039 45 | $y^x$ | 067 44 | SUM | 095 43 | RCL |
| 012 05 | 5 | 040 43 | RCL | 068 01 | 1 | 096 00 | 0 |
| 013 43 | RCL | 041 00 | 0 | 069 00 | 0 | 097 01 | 1 |
| 014 00 | 0 | 042 03 | 3 | 070 43 | RCL | 098 56 | *rtn |
| 015 04 | 4 | 043 65 | X | 071 00 | 0 | 099 46 | *LBL |
| 016 20 | *1/x | 044 43 | RCL | 072 04 | 4 | 100 12 | B |
| 017 65 | X | 045 00 | 0 | 073 95 | = | 101 90 | *if zro |
| 018 43 | RCL | 046 02 | 2 | 074 44 | SUM | 102 87 | *'1' |
| 019 00 | 0 | 047 95 | = | 075 01 | 1 | 103 42 | STO |
| 020 02 | 2 | 048 44 | SUM | 076 01 | 1 | 104 00 | 0 |
| 021 95 | = | 049 00 | 0 | 077 43 | RCL | 105 02 | 2 |
| 022 44 | SUM | 050 08 | 8 | 078 00 | 0 | 106 56 | *rtn |
| 023 00 | 0 | 051 43 | RCL | 079 02 | 2 | 107 46 | *LBL |
| 024 07 | 7 | 052 00 | 0 | 080 44 | SUM | 108 15 | E |
| 025 43 | RCL | 053 04 | 4 | 081 00 | 0 | 109 25 | CLR |
| 026 00 | 0 | 054 40 | *$x^2$ | 082 01 | 1 | 110 47 | *CMs |
| 027 04 | 4 | 055 65 | X | 083 43 | RCL | 111 01 | 1 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 | 05 Σf | 10 $\Sigma fx^3$ | 15 |
| 01 n | 06 $\Pi x$ | 11 $\Sigma fx^4$ | 16 |
| 02 f | 07 Σf/x | 12 Σfx/n | 17 |
| 03 t | 08 Σfx | 13 | 18 |
| 04 last x | 09 $\Sigma fx^2$ | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 42 | STO | 140 11 | A | 168 46 | *LBL | 196 47 | *'1' |
| 113 00 | 0 | 141 01 | 1 | 169 18 | *C' | 197 20 | *1/x |
| 114 02 | 2 | 142 94 | +/− | 170 43 | RCL | 198 56 | *rtn |
| 115 42 | STO | 143 22 | INV | 171 00 | 0 | 199 00 | |
| 116 00 | 0 | 144 49 | *PROD | 172 01 | 1 | 200 00 | |
| 117 06 | 6 | 145 00 | 0 | 173 55 | ÷ | 201 00 | |
| 118 46 | *LBL | 146 02 | 2 | 174 43 | RCL | 202 00 | |
| 119 13 | C | 147 43 | RCL | 175 00 | 0 | 203 00 | |
| 120 90 | *if zro | 148 00 | 0 | 176 07 | 7 | 204 00 | |
| 121 87 | *'1' | 149 01 | 1 | 177 95 | = | 205 00 | |
| 122 42 | STO | 150 56 | *rtn | 178 56 | *rtn | 206 00 | |
| 123 00 | 0 | 151 46 | *LBL | 179 46 | *LBL | 207 00 | |
| 124 03 | 3 | 152 16 | *A' | 180 19 | *D' | 208 00 | |
| 125 56 | *rtn | 153 43 | RCL | 181 43 | RCL | 209 00 | |
| 126 46 | *LBL | 154 01 | 1 | 182 00 | 0 | 210 00 | |
| 127 14 | D | 155 02 | 2 | 183 08 | 8 | 211 00 | |
| 128 42 | STO | 156 56 | *rtn | 184 55 | ÷ | 212 00 | |
| 129 00 | 0 | 157 46 | *LBL | 185 43 | RCL | 213 00 | |
| 130 04 | 4 | 158 17 | *B' | 186 00 | 0 | 214 00 | |
| 131 01 | 1 | 159 43 | RCL | 187 01 | 1 | 215 00 | |
| 132 94 | +/− | 160 00 | 0 | 188 95 | = | 216 00 | |
| 133 22 | INV | 161 06 | 6 | 189 35 | $\sqrt[x]{y}$ | 217 00 | |
| 134 49 | *PROD | 162 35 | $\sqrt[x]{y}$ | 190 43 | RCL | 218 00 | |
| 135 00 | 0 | 163 43 | RCL | 191 00 | 0 | 219 00 | |
| 136 02 | 2 | 164 00 | 0 | 192 03 | 3 | 220 00 | |
| 137 43 | RCL | 165 01 | 1 | 193 95 | = | 221 00 | |
| 138 00 | 0 | 166 95 | = | 194 56 | *rtn | 222 00 | |
| 139 04 | 4 | 167 56 | *rtn | 195 46 | *LBL | 223 00 | |

*Denotes 2nd function key

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 03 | 3 | 056 55 | ÷ | 084 09 | 9 |
| 001 11 | A | 029 65 | X | 057 43 | RCL | 085 55 | ÷ |
| 002 43 | RCL | 030 43 | RCL | 058 00 | 0 | 086 43 | RCL |
| 003 01 | 1 | 031 01 | 1 | 059 01 | 1 | 087 00 | 0 |
| 004 02 | 2 | 032 02 | 2 | 060 75 | − | 088 01 | 1 |
| 005 40 | *$x^2$ | 033 65 | X | 061 04 | 4 | 089 75 | − |
| 006 94 | +/− | 034 43 | RCL | 062 65 | X | 090 03 | 3 |
| 007 85 | + | 035 00 | 0 | 063 43 | RCL | 091 65 | X |
| 008 53 | ( | 036 09 | 9 | 064 01 | 1 | 092 43 | RCL |
| 009 43 | RCL | 037 55 | ÷ | 065 02 | 2 | 093 01 | 1 |
| 010 00 | 0 | 038 43 | RCL | 066 65 | X | 094 02 | 2 |
| 011 09 | 9 | 039 00 | 0 | 067 43 | RCL | 095 40 | *$x^2$ |
| 012 55 | ÷ | 040 01 | 1 | 068 01 | 1 | 096 40 | *$x^2$ |
| 013 43 | RCL | 041 85 | + | 069 00 | 0 | 097 95 | = |
| 014 00 | 0 | 042 43 | RCL | 070 55 | ÷ | 098 56 | *rtn |
| 015 01 | 1 | 043 01 | 1 | 071 43 | RCL | 099 46 | *LBL |
| 016 95 | = | 044 02 | 2 | 072 00 | 0 | 100 14 | D |
| 017 56 | *rtn | 045 65 | X | 073 01 | 1 | 101 11 | A |
| 018 46 | *LBL | 046 40 | *$x^2$ | 074 85 | + | 102 40 | *$x^2$ |
| 019 12 | B | 047 65 | X | 075 06 | 6 | 103 42 | STO |
| 020 43 | RCL | 048 02 | 2 | 076 65 | X | 104 01 | 1 |
| 021 01 | 1 | 049 95 | = | 077 43 | RCL | 105 03 | 3 |
| 022 00 | 0 | 050 56 | *rtn | 078 01 | 1 | 106 13 | C |
| 023 55 | ÷ | 051 46 | *LBL | 079 02 | 2 | 107 46 | *LBL |
| 024 43 | RCL | 052 13 | C | 080 40 | *$x^2$ | 108 87 | *'1' |
| 025 00 | 0 | 053 43 | RCL | 081 65 | X | 109 55 | ÷ |
| 026 01 | 1 | 054 01 | 1 | 082 43 | RCL | 110 43 | RCL |
| 027 75 | − | 055 01 | 1 | 083 00 | 0 | 111 01 | 1 |

*Denotes 2nd function key

### REGISTERS

| | | | |
|---|---|---|---|
| 00 | 05 $\Sigma f$ | 10 $\Sigma fx^3$ | 15 |
| 01 n | 06 $\Pi x'$ | 11 $\Sigma fx^4$ | 16 |
| 02 f | 07 $\Sigma f/x$ | 12 $\Sigma fx/n$ | 17 |
| 03 t | 08 $\Sigma fx'$ | 13 Used | 18 |
| 04 last x | 09 $\Sigma fx^2$ | 14 | 19 |

### FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 03 | 3 | 140 35 | $\sqrt[x]{y}$ | 168 43 | RCL | 196 00 | |
| 113 95 | = | 141 43 | RCL | 169 00 | 0 | 197 00 | |
| 114 56 | *rtn | 142 00 | 0 | 170 03 | 3 | 198 00 | |
| 115 46 | *LBL | 143 01 | 1 | 171 95 | = | 199 00 | |
| 116 15 | E | 144 95 | = | 172 56 | *rtn | 200 00 | |
| 117 11 | A | 145 56 | *rtn | 173 00 | | 201 00 | |
| 118 65 | X | 146 46 | *LBL | 174 00 | | 202 00 | |
| 119 40 | *$x^2$ | 147 18 | *'C' | 175 00 | | 203 00 | |
| 120 95 | = | 148 43 | RCL | 176 00 | | 204 00 | |
| 121 30 | *$\sqrt{x}$ | 149 00 | 0 | 177 00 | | 205 00 | |
| 122 95 | = | 150 01 | 1 | 178 00 | | 206 00 | |
| 123 42 | STO | 151 55 | ÷ | 179 00 | | 207 00 | |
| 124 01 | 1 | 152 43 | RCL | 180 00 | | 208 00 | |
| 125 03 | 3 | 153 00 | 0 | 181 00 | | 209 00 | |
| 126 12 | B | 154 07 | 7 | 182 00 | | 210 00 | |
| 127 41 | GTO | 155 95 | = | 183 00 | | 211 00 | |
| 128 87 | *'1' | 156 56 | *rtn | 184 00 | | 212 00 | |
| 129 46 | *LBL | 157 46 | *LBL | 185 00 | | 213 00 | |
| 130 16 | *'A' | 158 19 | *'D' | 186 00 | | 214 00 | |
| 131 43 | RCL | 159 43 | RCL | 187 00 | | 215 00 | |
| 132 01 | 1 | 160 00 | 0 | 188 00 | | 216 00 | |
| 133 02 | 2 | 161 08 | 8 | 189 00 | | 217 00 | |
| 134 56 | *rtn | 162 55 | ÷ | 190 00 | | 218 00 | |
| 135 46 | *LBL | 163 43 | RCL | 191 00 | | 219 00 | |
| 136 17 | *'B' | 164 00 | 0 | 192 00 | | 220 00 | |
| 137 43 | RCL | 165 01 | 1 | 193 00 | | 221 00 | |
| 138 00 | 0 | 166 95 | = | 194 00 | | 222 00 | |
| 139 06 | 6 | 167 35 | $\sqrt[x]{y}$ | 195 00 | | 223 00 | |

*Denotes 2nd function key

# LINEAR REGRESSION

The linear least-squares fit of input data points (x,y) is calculated using the following:

$$\text{Slope} = m = \frac{\frac{\Sigma x \Sigma y}{n} - \Sigma xy}{\frac{(\Sigma x)^2}{n} - \Sigma x^2}$$

$$\text{Intercept} = b = \frac{\Sigma y - m\Sigma x}{n}$$

The y′ for entered x and x′ for entered y are calculated as follows:

$$y' = mx + b$$

$$x' = (y - b)/m$$

The coefficient of determination is calculated as:

$$r^2 = m\left(\frac{\Sigma x \Sigma y}{n} - \Sigma xy\right) \Big/ \left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)$$

NOTES: 1. *The calculator must be initialized by pressing* 2nd **CMs** before each new set of data is entered.

2. The number of data points entered must be greater than 1 before any of the calculations are attempted.

3. This program may be used with the *Basic Statistics program* (ST1-01) to determine additional statistics. Data points entered on either program may be used for the other. Do not initialize if data from the other program is being used.

Example:

| x | 20.4 | 19.7 | 21.8 | 20.1 | 20.7 |
|---|---|---|---|---|---|
| y | 9.2 | 8.9 | 11.4 | 9.4 | 10.3 |

$r^2 = .9256678121$
$m = 1.22906793$
$b = -15.40505529$

For y = 12, x′ = 22.29742931
For x = 22.3, y′ = 12.00315956

TEXAS INSTRUMENTS

| ST1-08 | | | | ◆B◆ LINEAR REGRESSION |
|---|---|---|---|---|
| ◆A◆ LINEAR REGRESSION | | | | ST1-08 |
| y → x′ | x → y′ | | delete | |
| x | y | $r^2$ | slope | intcpt |

## USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | | |
| 2 | Initialize | | 2nd | **CMs** | | |
| 3 | Enter data | x | A | | | x |
| | Repeat 3 for each x,y | y | B | | | n |
| 4 | Delete data | | | | | |
| | a. Enter x (not necessary | x | A | | | x |
| | if deleting last pair | | | | | |
| | entered) | | | | | |
| | b. Enter y | y | 2nd | **D'** | | n − 1 |
| 5 | Calculate | | | | | |
| | a. $r^2$ | | C | | | $r^2$ |
| | b. slope | | D | | | m |
| | c. intercept | | E | | | b |
| | d. x′ | y | 2nd | **A'** | | x′ |
| | e. y′ | x | 2nd | **B'** | | y′ |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 01 | 1 | 056 43 | RCL | 084 00 | 0 |
| 001 19 | *D' | 029 94 | +/− | 057 00 | 0 | 085 01 | 1 |
| 002 95 | = | 030 44 | SUM | 058 00 | 0 | 086 44 | SUM |
| 003 94 | +/− | 031 00 | 0 | 059 56 | *rtn | 087 00 | 0 |
| 004 42 | STO | 032 03 | 3 | 060 46 | *LBL | 088 03 | 3 |
| 005 00 | 0 | 033 40 | *$x^2$ | 061 12 | B | 089 40 | *$x^2$ |
| 006 02 | 2 | 034 94 | +/− | 062 42 | STO | 090 44 | SUM |
| 007 44 | SUM | 035 44 | SUM | 063 00 | 0 | 091 00 | 0 |
| 008 00 | 0 | 036 00 | 0 | 064 02 | 2 | 092 06 | 6 |
| 009 04 | 4 | 037 06 | 6 | 065 44 | SUM | 093 01 | 1 |
| 010 40 | *$x^2$ | 038 01 | 1 | 066 00 | 0 | 094 41 | GTO |
| 011 94 | +/− | 039 94 | +/− | 067 04 | 4 | 095 88 | *2' |
| 012 44 | SUM | 040 46 | *LBL | 068 40 | *$x^2$ | 096 46 | *LBL |
| 013 00 | 0 | 041 88 | *2' | 069 44 | SUM | 097 14 | D |
| 014 07 | 7 | 042 44 | SUM | 070 00 | 0 | 098 43 | RCL |
| 015 43 | RCL | 043 00 | 0 | 071 07 | 7 | 099 00 | 0 |
| 016 00 | 0 | 044 00 | 0 | 072 43 | RCL | 100 03 | 3 |
| 017 02 | 2 | 045 43 | RCL | 073 00 | 0 | 101 55 | ÷ |
| 018 65 | X | 046 00 | 0 | 074 02 | 2 | 102 43 | RCL |
| 019 43 | RCL | 047 00 | 0 | 075 65 | X | 103 00 | 0 |
| 020 00 | 0 | 048 75 | − | 076 43 | RCL | 104 00 | 0 |
| 021 01 | 1 | 049 01 | 1 | 077 00 | 0 | 105 65 | X |
| 022 95 | = | 050 95 | = | 078 01 | 1 | 106 42 | STO |
| 023 44 | SUM | 051 30 | *$\sqrt{x}$ | 079 95 | = | 107 01 | 1 |
| 024 00 | 0 | 052 42 | STO | 080 44 | SUM | 108 00 | 0 |
| 025 05 | 5 | 053 00 | 0 | 081 00 | 0 | 109 43 | RCL |
| 026 43 | RCL | 054 08 | 8 | 082 05 | 5 | 110 00 | 0 |
| 027 00 | 0 | 055 25 | CLR | 083 43 | RCL | 111 04 | 4 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 n | 05 $\Sigma xy$ | 10 $\bar{x}$ | 15 |
| 01 x | 06 $\Sigma x^2$ | 11 m | 16 |
| 02 y | 07 $\Sigma y^2$ | 12 Used | 17 |
| 03 $\Sigma x$ | 08 $\sqrt{n-1}$ | 13 Used | 18 |
| 04 $\Sigma y$ | 09 | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | | 2 | | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 75 | − | 140 77 | *4' | 168 85 | + | 196 65 | X |
| 113 43 | RCL | 141 42 | STO | 169 43 | RCL | 197 43 | RCL |
| 114 00 | 0 | 142 01 | 1 | 170 01 | 1 | 198 01 | 1 |
| 115 05 | 5 | 143 02 | 2 | 171 02 | 2 | 199 03 | 3 |
| 116 95 | = | 144 46 | *LBL | 172 95 | = | 200 55 | ÷ |
| 117 94 | +/− | 145 15 | E | 173 55 | ÷ | 201 53 | ( |
| 118 42 | STO | 146 14 | D | 174 46 | *LBL | 202 43 | RCL |
| 119 01 | 1 | 147 65 | X | 175 87 | *1' | 203 00 | 0 |
| 120 03 | 3 | 148 43 | RCL | 176 43 | RCL | 204 04 | 4 |
| 121 55 | ÷ | 149 00 | 0 | 177 01 | 1 | 205 40 | *$x^2$ |
| 122 53 | ( | 150 03 | 3 | 178 01 | 1 | 206 55 | ÷ |
| 123 43 | RCL | 151 94 | +/− | 179 95 | = | 207 43 | RCL |
| 124 00 | 0 | 152 85 | + | 180 56 | *rtn | 208 00 | 0 |
| 125 06 | 6 | 153 43 | RCL | 181 46 | *LBL | 209 00 | 0 |
| 126 75 | − | 154 00 | 0 | 182 17 | *B' | 210 94 | +/− |
| 127 43 | RCL | 155 04 | 4 | 183 51 | SBR | 211 85 | + |
| 128 00 | 0 | 156 95 | = | 184 77 | *4' | 212 43 | RCL |
| 129 03 | 3 | 157 55 | ÷ | 185 85 | + | 213 00 | 0 |
| 130 65 | X | 158 43 | RCL | 186 53 | ( | 214 07 | 7 |
| 131 43 | RCL | 159 00 | 0 | 187 43 | RCL | 215 95 | = |
| 132 01 | 1 | 160 00 | 0 | 188 01 | 1 | 216 56 | *rtn |
| 133 00 | 0 | 161 95 | = | 189 02 | 2 | 217 46 | *LBL |
| 134 95 | = | 162 56 | *rtn | 190 65 | X | 218 11 | A |
| 135 42 | STO | 163 46 | *LBL | 191 41 | GTO | 219 42 | STO |
| 136 01 | 1 | 164 16 | *A' | 192 87 | *1' | 220 00 | 0 |
| 137 01 | 1 | 165 51 | SBR | 193 46 | *LBL | 221 01 | 1 |
| 138 56 | *rtn | 166 77 | *4' | 194 13 | C | 222 56 | *rtn |
| 139 46 | *LBL | 167 94 | +/− | 195 14 | D | 223 00 | |

*Denotes 2nd function key

## POWER CURVE FIT

Given a set of data points:

$$\{x_i, y_i\},\ i = 1, 2, \ldots, n\}$$

Where: $x_i > 0$ and $y_i > 0$, *this program will fit a power curve*:

$$y = ax^b \qquad (a > 0)$$

The problem may be interpreted as a linear regression problem by using the equation in the form:

$$\ln y = b \ln x + \ln a$$

The computed statistics are:

1. Regression coefficients

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - \frac{b\, \Sigma \ln x_i}{n}\right]$$

$$b = \frac{\Sigma(\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma(\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}}$$

NOTE: n is a positive integer ≠ 1.

2. Correlation coefficient

$$r^2 = \frac{\left[\Sigma (\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}\right]^2}{\left[\Sigma (\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}\right]\left[\Sigma (\ln y_i)^2 - \dfrac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value x′ for given y

$$x' = \sqrt[b]{y/a}$$

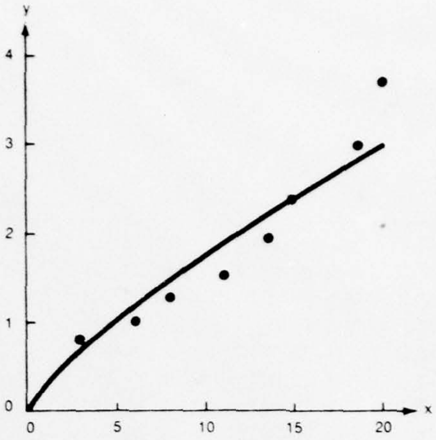
4. Estimated value y′ for given x

$$y' = ax^b$$

Example:

| $x_i$ | 3 | 6 | 8 | 11 | 13.5 | 15 | 18.5 | 20 |
|---|---|---|---|---|---|---|---|---|
| $y_i$ | .8 | 1.12 | 1.3 | 1.53 | 1.95 | 2.4 | 3.0 | 3.7 |

a = .2901902009
b = .7759169351
$r^2$ = 0.921727263
for x = 9 , y′ = 1.596230726
for y = 2.8, x′ = 18.56904361



TEXAS INSTRUMENTS

◆B■ POWER CURVE FIT ST1-09

◆A■ POWER CURVE FIT ST1-09

| | | | | |
|---|---|---|---|---|
| delete | y → x′ | x → y′ | b | |
| INIT | $x_i$ | $y_i$ | a | $r^2$ |

### USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | | |
| 2 | Initialize | | A | | | 0 |
| 3 | Enter data | $x_i$ | B | | | $x_i$ |
| | (repeat for all i) | $y_i$ | C | | | n |
| 4 | Delete data | | | | | |
| | a. Enter x | x | B | | | x |
| | b. Enter y | y | 2nd | A′ | | n − 1 |
| 5 | Calculate regression | | D | | | a |
| | coefficients | | 2nd | D′ | | b |
| 6 | Calculate correlation | | E | | | $r^2$ |
| | coefficient | | | | | |
| 7 | Compute estimated | | | | | |
| | value for | | | | | |
| | x′ given y | y | 2nd | B′ | | x′ |
| | y′ given x | x | 2nd | C′ | | y′ |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 00 | 0 | 056 46 | *LBL | 084 00 | 0 |
| 001 11 | A | 029 06 | 6 | 057 16 | *'A' | 085 02 | 2 |
| 002 47 | *CMs | 030 44 | SUM | 058 23 | lnx | 086 43 | RCL |
| 003 25 | CLR | 031 00 | 0 | 059 42 | STO | 087 00 | 0 |
| 004 56 | *rtn | 032 01 | 1 | 060 00 | 0 | 088 07 | 7 |
| 005 46 | *LBL | 033 49 | *PROD | 061 07 | 7 | 089 22 | INV |
| 006 12 | B | 034 00 | 0 | 062 22 | INV | 090 44 | SUM |
| 007 23 | lnx | 035 07 | 7 | 063 44 | SUM | 091 00 | 0 |
| 008 42 | STO | 036 40 | *$x^2$ | 064 00 | 0 | 092 04 | 4 |
| 009 00 | 0 | 037 44 | SUM | 065 03 | 3 | 093 01 | 1 |
| 010 06 | 6 | 038 00 | 0 | 066 40 | *$x^2$ | 094 94 | +/− |
| 011 22 | INV | 039 02 | 2 | 067 22 | INV | 095 41 | GTO |
| 012 23 | lnx | 040 43 | RCL | 068 44 | SUM | 096 87 | *'1' |
| 013 56 | *rtn | 041 00 | 0 | 069 00 | 0 | 097 46 | *LBL |
| 014 46 | *LBL | 042 07 | 7 | 070 05 | 5 | 098 14 | D |
| 015 13 | C | 043 44 | SUM | 071 43 | RCL | 099 43 | RCL |
| 016 23 | lnx | 044 00 | 0 | 072 00 | 0 | 100 00 | 0 |
| 017 42 | STO | 045 04 | 4 | 073 06 | 6 | 101 02 | 2 |
| 018 00 | 0 | 046 01 | 1 | 074 22 | INV | 102 75 | − |
| 019 07 | 7 | 047 46 | *LBL | 075 44 | SUM | 103 43 | RCL |
| 020 44 | SUM | 048 87 | *'1' | 076 00 | 0 | 104 00 | 0 |
| 021 00 | 0 | 049 44 | SUM | 077 01 | 1 | 105 01 | 1 |
| 022 03 | 3 | 050 00 | 0 | 078 49 | *PROD | 106 40 | *$x^2$ |
| 023 40 | *$x^2$ | 051 00 | 0 | 079 00 | 0 | 107 55 | ÷ |
| 024 44 | SUM | 052 43 | RCL | 080 07 | 7 | 108 43 | RCL |
| 025 00 | 0 | 053 00 | 0 | 081 40 | *$x^2$ | 109 00 | 0 |
| 026 05 | 5 | 054 00 | 0 | 082 22 | INV | 110 00 | 0 |
| 027 43 | RCL | 055 56 | *rtn | 083 44 | SUM | 111 95 | = |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 n | 05 $\Sigma(\ln y)^2$ | 10 Used | 15 |
| 01 $\Sigma \ln x$ | 06 lnx | 11 Used | 16 |
| 02 $\Sigma(\ln x)^2$ | 07 lny | 12 $r^2$ | 17 |
| 03 $\Sigma \ln y$ | 08 b | 13 | 18 |
| 04 $\Sigma(\ln x)(\ln y)$ | 09 a | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 20 | *1/x | 140 08 | 8 | 168 46 | *LBL | 196 01 | 1 |
| 113 42 | STO | 141 94 | +/− | 169 17 | *'B' | 197 00 | 0 |
| 114 01 | 1 | 142 65 | X | 170 55 | ÷ | 198 65 | X |
| 115 00 | 0 | 143 43 | RCL | 171 43 | RCL | 199 53 | ( |
| 116 65 | X | 144 00 | 0 | 172 00 | 0 | 200 43 | RCL |
| 117 53 | ( | 145 01 | 1 | 173 09 | 9 | 201 00 | 0 |
| 118 43 | RCL | 146 85 | + | 174 95 | = | 202 05 | 5 |
| 119 00 | 0 | 147 43 | RCL | 175 35 | $\sqrt[x]{y}$ | 203 75 | − |
| 120 04 | 4 | 148 00 | 0 | 176 43 | RCL | 204 43 | RCL |
| 121 75 | − | 149 03 | 3 | 177 00 | 0 | 205 00 | 0 |
| 122 43 | RCL | 150 95 | = | 178 08 | 8 | 206 03 | 3 |
| 123 00 | 0 | 151 55 | ÷ | 179 95 | = | 207 40 | *$x^2$ |
| 124 01 | 1 | 152 43 | RCL | 180 56 | *rtn | 208 55 | ÷ |
| 125 65 | X | 153 00 | 0 | 181 46 | *LBL | 209 43 | RCL |
| 126 43 | RCL | 154 00 | 0 | 182 18 | *'C' | 210 00 | 0 |
| 127 00 | 0 | 155 95 | = | 183 45 | $y^x$ | 211 00 | 0 |
| 128 03 | 3 | 156 22 | INV | 184 43 | RCL | 212 54 | ) |
| 129 55 | ÷ | 157 23 | lnx | 185 00 | 0 | 213 20 | *1/x |
| 130 43 | RCL | 158 42 | STO | 186 08 | 8 | 214 65 | X |
| 131 00 | 0 | 159 00 | 0 | 187 65 | X | 215 43 | RCL |
| 132 00 | 0 | 160 09 | 9 | 188 43 | RCL | 216 01 | 1 |
| 133 54 | ) | 161 56 | *rtn | 189 00 | 0 | 217 01 | 1 |
| 134 42 | STO | 162 46 | *LBL | 190 09 | 9 | 218 40 | *$x^2$ |
| 135 01 | 1 | 163 19 | *'D' | 191 95 | = | 219 95 | = |
| 136 01 | 1 | 164 43 | RCL | 192 56 | *rtn | 220 42 | STO |
| 137 95 | = | 165 00 | 0 | 193 46 | *LBL | 221 01 | 1 |
| 138 42 | STO | 166 08 | 8 | 194 15 | E | 222 02 | 2 |
| 139 00 | 0 | 167 56 | *rtn | 195 43 | RCL | 223 56 | *rtn |

*Denotes 2nd function key

# EXPONENTIAL CURVE FIT

Given a set of data points:

$$\{(x_i, y_i), i = 1, 2, \ldots, n\} \qquad y_i > 0$$

This program finds the least-squares fit for an exponential function of the form:

$$y = ae^{bx} \ (a > 0)$$

The problem is computed according to the linear equation

$$\ln y = \ln a + bx$$

Statistical outputs:

1. Coefficients a, b

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma x_i}{n}\right]$$

$$b = \frac{\Sigma x_i \ln y_i - \frac{\Sigma x_i \Sigma \ln y_i}{n}}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}$$

Note: n is positive integer $\neq 1$.

2. Correlation coefficient

$$r^2 = \frac{\left[\Sigma x_i \ln y_i - \frac{\Sigma x_i \Sigma \ln y_i}{n}\right]^2}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma (\ln y_i)^2 - \frac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value for x' given y

$$x' = \frac{1}{b}\ln\left(\frac{y}{a}\right)$$

4. Estimated value for y' given x

$$y' = ae^{bx}$$

Example:

| $x_i$ | 1.42 | 1.94 | 2.63 | 3.11 | 3.7 | 4.25 |
|---|---|---|---|---|---|---|
| $y_i$ | .63 | .9 | 1.42 | 1.9 | 2.43 | 2.89 |

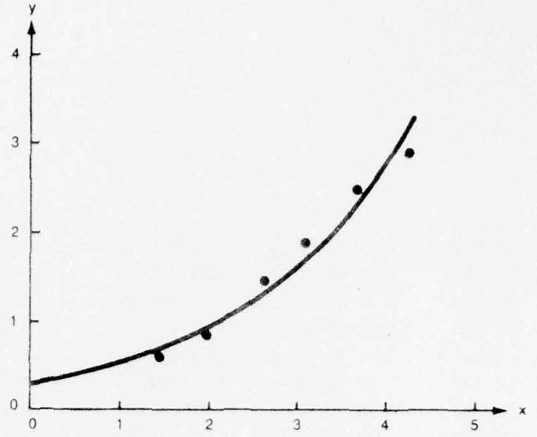
a = .3131935086

b = .5477927735

$r^2$ = .9810626986

For x = 1.6, y' = .7524162355

For y = 2.2, x' = 3.558629277



TEXAS INSTRUMENTS

ST1-10 EXPONENTIAL CURVE FIT ◆B◆

◆A◆ EXPONENTIAL CURVE FIT ST1-10

| | | | | |
|---|---|---|---|---|
| delete | $y \rightarrow x'$ | $x \rightarrow y'$ | b | |
| INIT | $x_i$ | $y_i$ | a | $r^2$ |

## USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | | |
| 2 | Initialize | | A | | | 0. |
| 3 | Enter data | $x_i$ | B | | | $x_i$ |
| | (repeat for all i) | $y_i$ | C | | | n |
| 4 | Delete data | | | | | |
| | a. Enter x | x | B | | | x |
| | b. Enter y | y | 2nd | A' | | n−1 |
| 5 | Calculate coefficients | | D | | | a |
| | | | 2nd | D' | | b |
| 6 | Calculate correlation | | | | | |
| | coefficient | | E | | | $r^2$ |
| 7 | Compute estimated | | | | | |
| | value for | | | | | |
| | x' given y | y | 2nd | B' | | x' |
| | y' given x | x | 2nd | C' | | y' |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 00 | 0 | 056 56 | *rtn | 084 44 | SUM |
| 001 10 | *E' | 029 05 | 5 | 057 46 | *LBL | 085 00 | 0 |
| 002 43 | RCL | 030 43 | RCL | 058 16 | *A' | 086 02 | 2 |
| 003 00 | 0 | 031 00 | 0 | 059 23 | lnx | 087 43 | RCL |
| 004 00 | 0 | 032 06 | 6 | 060 42 | STO | 088 00 | 0 |
| 005 56 | *rtn | 033 44 | SUM | 061 00 | 0 | 089 07 | 7 |
| 006 46 | *LBL | 034 00 | 0 | 062 07 | 7 | 090 22 | INV |
| 007 11 | A | 035 01 | 1 | 063 22 | INV | 091 44 | SUM |
| 008 47 | *CMs | 036 49 | *PROD | 064 44 | SUM | 092 00 | 0 |
| 009 25 | CLR | 037 00 | 0 | 065 00 | 0 | 093 04 | 4 |
| 010 56 | *rtn | 038 07 | 7 | 066 03 | 3 | 094 01 | 1 |
| 011 46 | *LBL | 039 40 | *$x^2$ | 067 40 | *$x^2$ | 095 94 | +/− |
| 012 12 | B | 040 44 | SUM | 068 22 | INV | 096 41 | GTO |
| 013 42 | STO | 041 00 | 0 | 069 44 | SUM | 097 87 | *1' |
| 014 00 | 0 | 042 02 | 2 | 070 00 | 0 | 098 46 | *LBL |
| 015 06 | 6 | 043 43 | RCL | 071 05 | 5 | 099 14 | D |
| 016 56 | *rtn | 044 00 | 0 | 072 43 | RCL | 100 43 | RCL |
| 017 46 | *LBL | 045 07 | 7 | 073 00 | 0 | 101 00 | 0 |
| 018 13 | C | 046 44 | SUM | 074 06 | 6 | 102 02 | 2 |
| 019 23 | lnx | 047 00 | 0 | 075 22 | INV | 103 75 | − |
| 020 42 | STO | 048 04 | 4 | 076 44 | SUM | 104 43 | RCL |
| 021 00 | 0 | 049 01 | 1 | 077 00 | 0 | 105 00 | 0 |
| 022 07 | 7 | 050 46 | *LBL | 078 01 | 1 | 106 01 | 1 |
| 023 44 | SUM | 051 87 | *1' | 079 49 | *PROD | 107 40 | *$x^2$ |
| 024 00 | 0 | 052 44 | SUM | 080 00 | 0 | 108 55 | ÷ |
| 025 03 | 3 | 053 00 | 0 | 081 07 | 7 | 109 10 | *E' |
| 026 40 | *$x^2$ | 054 00 | 0 | 082 40 | *$x^2$ | 110 95 | = |
| 027 44 | SUM | 055 10 | *E' | 083 22 | INV | 111 20 | *1/x |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 n | 05 $\Sigma(\ln y)^2$ | 10 Used | 15 |
| 01 $\Sigma x$ | 06 x | 11 Used | 16 |
| 02 $\Sigma x^2$ | 07 x lny | 12 $r^2$ | 17 |
| 03 $\Sigma \ln y$ | 08 b | 13 | 18 |
| 04 $\Sigma x \ln y$ | 09 a | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 42 | STO | 140 43 | RCL | 168 09 | 9 | 196 00 | 0 |
| 113 01 | 1 | 141 00 | 0 | 169 95 | = | 197 65 | X |
| 114 00 | 0 | 142 01 | 1 | 170 23 | lnx | 198 53 | ( |
| 115 65 | X | 143 85 | + | 171 55 | ÷ | 199 43 | RCL |
| 116 53 | ( | 144 43 | RCL | 172 43 | RCL | 200 00 | 0 |
| 117 43 | RCL | 145 00 | 0 | 173 00 | 0 | 201 05 | 5 |
| 118 00 | 0 | 146 03 | 3 | 174 08 | 8 | 202 75 | − |
| 119 04 | 4 | 147 95 | = | 175 95 | = | 203 43 | RCL |
| 120 75 | − | 148 55 | ÷ | 176 56 | *rtn | 204 00 | 0 |
| 121 43 | RCL | 149 10 | *E' | 177 46 | *LBL | 205 03 | 3 |
| 122 00 | 0 | 150 95 | = | 178 18 | *C' | 206 40 | *$x^2$ |
| 123 01 | 1 | 151 22 | INV | 179 65 | X | 207 55 | ÷ |
| 124 65 | X | 152 23 | lnx | 180 43 | RCL | 208 10 | *E' |
| 125 43 | RCL | 153 42 | STO | 181 00 | 0 | 209 54 | ) |
| 126 00 | 0 | 154 00 | 0 | 182 08 | 8 | 210 20 | *1/x |
| 127 03 | 3 | 155 09 | 9 | 183 95 | = | 211 65 | X |
| 128 55 | ÷ | 156 56 | *rtn | 184 22 | INV | 212 43 | RCL |
| 129 10 | *E' | 157 46 | *LBL | 185 23 | lnx | 213 01 | 1 |
| 130 54 | ) | 158 19 | *D' | 186 65 | X | 214 01 | 1 |
| 131 42 | STO | 159 43 | RCL | 187 43 | RCL | 215 40 | *$x^2$ |
| 132 01 | 1 | 160 00 | 0 | 188 00 | 0 | 216 95 | = |
| 133 01 | 1 | 161 08 | 8 | 189 09 | 9 | 217 42 | STO |
| 134 95 | = | 162 56 | *rtn | 190 95 | = | 218 01 | 1 |
| 135 42 | STO | 163 46 | *LBL | 191 56 | *rtn | 219 02 | 2 |
| 136 00 | 0 | 164 17 | *B' | 192 46 | *LBL | 220 56 | *rtn |
| 137 08 | 8 | 165 55 | ÷ | 193 15 | E | 221 00 | |
| 138 94 | +/− | 166 43 | RCL | 194 43 | RCL | 222 00 | |
| 139 65 | X | 167 00 | 0 | 195 01 | 1 | 223 00 | |

*Denotes 2nd function key

## LOGARITHMIC CURVE FIT

Given a set of data points:

$$\{(x_i, y_i), i = 1, 2, \ldots, n\}$$

**Where**: $x_i > 0$, this program fits a logarithmic curve:

$$y = a + b\ln x$$

*Statistical outputs*:

1. Regression coefficients

$$a = \frac{\Sigma y_i - b\Sigma \ln x_i}{n}$$

$$b = \frac{\Sigma y_i \ln x_i - \frac{\Sigma \ln x_i \Sigma y_i}{n}}{\Sigma(\ln x_i)^2 - \frac{\Sigma(\ln x_i)^2}{n}}$$

2. Correlation coefficient

$$r^2 = \frac{\left[\Sigma y_i \ln x_i - \frac{\Sigma \ln x_i \Sigma y_i}{n}\right]^2}{\left[\Sigma(\ln x_i)^2 - \frac{\Sigma(\ln x_i)^2}{n}\right]\left[\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right]}$$

3. Estimated value for $x'$ given y

$$x' = \exp\left[\frac{y - a}{b}\right]$$

4. Estimated value for $y'$ given x

$$y' = a + b\ln x$$

NOTE: n is positive integer $\geq 1$.

Example:

| $x_i$ | 2 | 5 | 7 | 11 | 13 | 14 |
|---|---|---|---|---|---|---|
| $y_i$ | 5.3 | 10.8 | 17.5 | 28.2 | 36.9 | 44.4 |

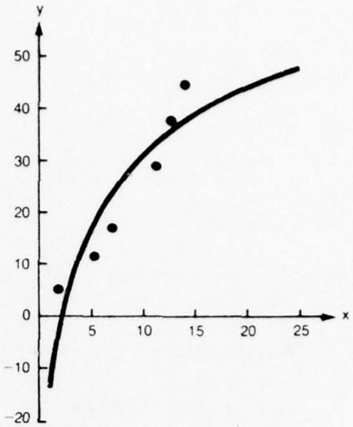
$a = -13.49953513$

$b = 18.91052316$

$r^2 = .8418448418$

For x = 21. $y' = 44.07397695$

For y = 40. $x' = 16.93001448$



TEXAS INSTRUMENTS

| | | | | |
|---|---|---|---|---|
| ◆B◆ LOGARITHMIC CURVE FIT | | | | ST1-11 |
| ◆A◆ LOGARITHMIC CURVE FIT | | | | ST1-11 |
| delete | $y \rightarrow x'$ | $x \rightarrow y'$ | b | |
| *INIT* | $x_i$ | $y_i$ | a | $r^2$ |

### USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | | DISPLAY |
|---|---|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | | | |
| 2 | Initialize | | A | | | | 0. |
| 3 | Enter data | $x_i$ | B | | | | $x_i$ |
| | (repeat for all i) | $y_i$ | C | | | | n |
| 4 | Delete data | | | | | | |
| | a. Enter x | x | B | | | | x |
| | b. Enter y | y | 2nd | A' | | | n−1 |
| 5 | Calculate coefficients | | D | | | | a |
| | | | 2nd | D' | | | b |
| 6 | Calculate correlation | | | | | | |
| | coefficient | | E | | | | $r^2$ |
| 7 | Compute estimated | | | | | | |
| | value for | | | | | | |
| | $x'$ given y | y | 2nd | B' | | | $x'$ |
| | $y'$ given x | x | 2nd | C' | | | $y'$ |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 00 | 0 | 056 00 | 0 | 084 00 | 0 |
| 001 11 | A | 029 01 | 1 | 057 07 | 7 | 085 07 | 7 |
| 002 47 | *CMs | 030 49 | *PROD | 058 22 | INV | 086 22 | INV |
| 003 25 | CLR | 031 00 | 0 | 059 44 | SUM | 087 44 | SUM |
| 004 56 | *rtn | 032 07 | 7 | 060 00 | 0 | 088 00 | 0 |
| 005 46 | *LBL | 033 40 | *$x^2$ | 061 03 | 3 | 089 04 | 4 |
| 006 12 | B | 034 44 | SUM | 062 40 | *$x^2$ | 090 01 | 1 |
| 007 42 | STO | 035 00 | 0 | 063 22 | INV | 091 94 | +/− |
| 008 00 | 0 | 036 02 | 2 | 064 44 | SUM | 092 41 | GTO |
| 009 06 | 6 | 037 43 | RCL | 065 00 | 0 | 093 87 | *'1' |
| 010 56 | *rtn | 038 00 | 0 | 066 05 | 5 | 094 46 | *LBL |
| 011 46 | *LBL | 039 07 | 7 | 067 43 | RCL | 095 14 | D |
| 012 13 | C | 040 44 | SUM | 068 00 | 0 | 096 43 | RCL |
| 013 42 | STO | 041 00 | 0 | 069 06 | 6 | 097 00 | 0 |
| 014 00 | 0 | 042 04 | 4 | 070 23 | lnx | 098 02 | 2 |
| 015 07 | 7 | 043 01 | 1 | 071 22 | INV | 099 75 | − |
| 016 44 | SUM | 044 46 | *LBL | 072 44 | SUM | 100 43 | RCL |
| 017 00 | 0 | 045 87 | *'1' | 073 00 | 0 | 101 00 | 0 |
| 018 03 | 3 | 046 44 | SUM | 074 01 | 1 | 102 01 | 1 |
| 019 40 | *$x^2$ | 047 00 | 0 | 075 49 | *PROD | 103 40 | *$x^2$ |
| 020 44 | SUM | 048 00 | 0 | 076 00 | 0 | 104 55 | ÷ |
| 021 00 | 0 | 049 43 | RCL | 077 07 | 7 | 105 43 | RCL |
| 022 05 | 5 | 050 00 | 0 | 078 40 | *$x^2$ | 106 00 | 0 |
| 023 43 | RCL | 051 00 | 0 | 079 22 | INV | 107 00 | 0 |
| 024 00 | 0 | 052 56 | *rtn | 080 44 | SUM | 108 95 | = |
| 025 06 | 6 | 053 46 | *LBL | 081 00 | 0 | 109 20 | *1/x |
| 026 23 | lnx | 054 16 | *A' | 082 02 | 2 | 110 42 | STO |
| 027 44 | SUM | 055 42 | STO | 083 43 | RCL | 111 01 | 1 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 n | 05 $\Sigma y^2$ | 10 Used | 15 |
| 01 $\Sigma$lnx | 06 x | 11 Used | 16 |
| 02 $\Sigma(\ln x)^2$ | 07 y | 12 $r^2$ | 17 |
| 03 $\Sigma$y | 08 b | 13 | 18 |
| 04 $\Sigma$y lnx | 09 a | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 00 | 0 | 140 43 | RCL | 168 65 | X | 196 75 | − |
| 113 65 | X | 141 00 | 0 | 169 53 | ( | 197 43 | RCL |
| 114 53 | ( | 142 01 | 1 | 170 43 | RCL | 198 00 | 0 |
| 115 43 | RCL | 143 85 | + | 171 00 | 0 | 199 09 | 9 |
| 116 00 | 0 | 144 43 | RCL | 172 05 | 5 | 200 95 | = |
| 117 04 | 4 | 145 00 | 0 | 173 75 | − | 201 55 | ÷ |
| 118 75 | − | 146 03 | 3 | 174 43 | RCL | 202 43 | RCL |
| 119 43 | RCL | 147 95 | = | 175 00 | 0 | 203 00 | 0 |
| 120 00 | 0 | 148 55 | ÷ | 176 03 | 3 | 204 08 | 8 |
| 121 01 | 1 | 149 43 | RCL | 177 40 | *$x^2$ | 205 95 | = |
| 122 65 | X | 150 00 | 0 | 178 55 | ÷ | 206 22 | INV |
| 123 43 | RCL | 151 00 | 0 | 179 43 | RCL | 207 23 | lnx |
| 124 00 | 0 | 152 95 | = | 180 00 | 0 | 208 56 | *rtn |
| 125 03 | 3 | 153 42 | STO | 181 00 | 0 | 209 46 | *LBL |
| 126 55 | ÷ | 154 00 | 0 | 182 54 | ) | 210 18 | *C' |
| 127 43 | RCL | 155 09 | 9 | 183 20 | *1/x | 211 23 | lnx |
| 128 00 | 0 | 156 56 | *rtn | 184 65 | X | 212 65 | X |
| 129 00 | 0 | 157 46 | *LBL | 185 43 | RCL | 213 43 | RCL |
| 130 54 | ) | 158 19 | *D' | 186 01 | 1 | 214 00 | 0 |
| 131 42 | STO | 159 43 | RCL | 187 01 | 1 | 215 08 | 8 |
| 132 01 | 1 | 160 00 | 0 | 188 40 | *$x^2$ | 216 85 | + |
| 133 01 | 1 | 161 08 | 8 | 189 95 | = | 217 43 | RCL |
| 134 95 | = | 162 56 | *rtn | 190 42 | STO | 218 00 | 0 |
| 135 42 | STO | 163 46 | *LBL | 191 01 | 1 | 219 09 | 9 |
| 136 00 | 0 | 164 15 | E | 192 02 | 2 | 220 95 | = |
| 137 08 | 8 | 165 43 | RCL | 193 56 | *rtn | 221 56 | *rtn |
| 138 94 | +/− | 166 01 | 1 | 194 46 | *LBL | 222 00 | |
| 139 65 | X | 167 00 | 0 | 195 17 | *B' | 223 00 | |

*Denotes 2nd function key

## NORMAL DISTRIBUTION

In order to calculate the standard normal distribution, the following probability function is used:

$$Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Then: $Q(x) = Z(x)(b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5) + \epsilon(x)$

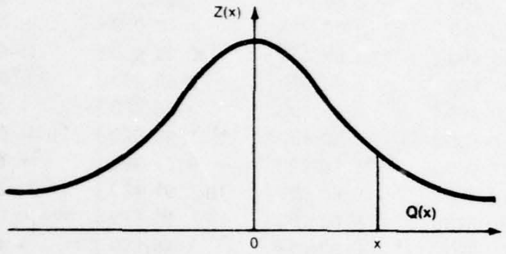
Where: $t = \frac{1}{1 + px}$

$|\epsilon(x)| < 7.5 \times 10^{-8}$
$p = .2316419$
$b_1 = .319381530$
$b_2 = -.356563782$
$b_3 = 1.781477937$
$b_4 = -1.821255978$
$b_5 = 1.330274429$

NOTES:
1. $Z(-x) = Z(x)$, $P(x) = 1 - Q(x)$
2. $|x| < 21.25$
3. Z(x) must be calculated before Q(x)

Reference: *Handbook of Mathematical Functions*, National Bureau of Standards, 1964

Example: For $x = 2.02$, $Z(x) = .0518635767$, $Q(x) = .0216916245$



TEXAS INSTRUMENTS

ST1-15 ◆B■ NORMAL DISTRIBUTION

◆A■ NORMAL DISTRIBUTION ST1-15

| | | | | |
|---|---|---|---|---|
| x → Z(x) | Q(x) | | | |

### USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | | |
| 2 | Enter data | x | A | | | Z(x) |
| 3 | Calculate Q(x) | | B | | | Q(x) |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 30 | *$\sqrt{x}$ | 056 65 | X | 084 43 | RCL |
| 001 11 | A | 029 95 | = | 057 93 | • | 085 00 | 0 |
| 002 22 | INV | 030 42 | STO | 058 03 | 3 | 086 02 | 2 |
| 003 50 | *st flg | 031 00 | 0 | 059 01 | 1 | 087 45 | $y^x$ |
| 004 01 | 1 | 032 01 | 1 | 060 09 | 9 | 088 03 | 3 |
| 005 80 | *if pos | 033 56 | *rtn | 061 03 | 3 | 089 65 | X |
| 006 87 | *'1' | 034 46 | *LBL | 062 08 | 8 | 090 01 | 1 |
| 007 94 | +/− | 035 12 | B | 063 01 | 1 | 091 93 | • |
| 008 50 | *st flg | 036 25 | CLR | 064 05 | 5 | 092 07 | 7 |
| 009 01 | 1 | 037 93 | • | 065 03 | 3 | 093 08 | 8 |
| 010 46 | *LBL | 038 02 | 2 | 066 75 | − | 094 01 | 1 |
| 011 87 | *'1' | 039 03 | 3 | 067 43 | RCL | 095 04 | 4 |
| 012 42 | STO | 040 01 | 1 | 068 00 | 0 | 096 07 | 7 |
| 013 00 | 0 | 041 06 | 6 | 069 02 | 2 | 097 07 | 7 |
| 014 00 | 0 | 042 04 | 4 | 070 45 | $y^x$ | 098 09 | 9 |
| 015 40 | *$x^2$ | 043 01 | 1 | 071 02 | 2 | 099 03 | 3 |
| 016 55 | ÷ | 044 09 | 9 | 072 65 | X | 100 07 | 7 |
| 017 02 | 2 | 045 65 | X | 073 93 | • | 101 75 | − |
| 018 95 | = | 046 43 | RCL | 074 03 | 3 | 102 43 | RCL |
| 019 94 | +/− | 047 00 | 0 | 075 05 | 5 | 103 00 | 0 |
| 020 22 | INV | 048 00 | 0 | 076 06 | 6 | 104 02 | 2 |
| 021 23 | lnx | 049 85 | + | 077 05 | 5 | 105 45 | $y^x$ |
| 022 55 | ÷ | 050 01 | 1 | 078 06 | 6 | 106 04 | 4 |
| 023 53 | ( | 051 95 | = | 079 03 | 3 | 107 65 | X |
| 024 59 | *$\pi$ | 052 20 | *1/x | 080 07 | 7 | 108 01 | 1 |
| 025 65 | X | 053 42 | STO | 081 08 | 8 | 109 93 | • |
| 026 02 | 2 | 054 00 | 0 | 082 02 | 2 | 110 08 | 8 |
| 027 54 | ) | 055 02 | 2 | 083 85 | + | 111 02 | 2 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 Used | 05 | 10 | 15 |
| 01 Used | 06 | 11 | 16 |
| 02 Used | 07 | 12 | 17 |
| 03 | 08 | 13 | 18 |
| 04 | 09 | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 Used | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 01 | 1 | 140 00 | 0 | 168 00 | | 196 00 | |
| 113 02 | 2 | 141 01 | 1 | 169 00 | | 197 00 | |
| 114 05 | 5 | 142 95 | = | 170 00 | | 198 00 | |
| 115 05 | 5 | 143 60 | *if flg | 171 00 | | 199 00 | |
| 116 09 | 9 | 144 01 | 1 | 172 00 | | 200 00 | |
| 117 07 | 7 | 145 69 | *'9' | 173 00 | | 201 00 | |
| 118 08 | 8 | 146 56 | *rtn | 174 00 | | 202 00 | |
| 119 85 | + | 147 46 | *LBL | 175 00 | | 203 00 | |
| 120 43 | RCL | 148 69 | *'9' | 176 00 | | 204 00 | |
| 121 00 | 0 | 149 94 | +/− | 177 00 | | 205 00 | |
| 122 02 | 2 | 150 85 | + | 178 00 | | 206 00 | |
| 123 45 | $y^x$ | 151 01 | 1 | 179 00 | | 207 00 | |
| 124 05 | 5 | 152 95 | = | 180 00 | | 208 00 | |
| 125 65 | X | 153 56 | *rtn | 181 00 | | 209 00 | |
| 126 01 | 1 | 154 00 | | 182 00 | | 210 00 | |
| 127 93 | • | 155 00 | | 183 00 | | 211 00 | |
| 128 03 | 3 | 156 00 | | 184 00 | | 212 00 | |
| 129 03 | 3 | 157 00 | | 185 00 | | 213 00 | |
| 130 00 | 0 | 158 00 | | 186 00 | | 214 00 | |
| 131 02 | 2 | 159 00 | | 187 00 | | 215 00 | |
| 132 07 | 7 | 160 00 | | 188 00 | | 216 00 | |
| 132 04 | 4 | 161 00 | | 189 00 | | 217 00 | |
| 134 04 | 4 | 162 00 | | 190 00 | | 218 00 | |
| 135 02 | 2 | 163 00 | | 191 00 | | 219 00 | |
| 136 09 | 9 | 164 00 | | 192 00 | | 220 00 | |
| 137 95 | = | 165 00 | | 193 00 | | 221 00 | |
| 138 65 | X | 166 00 | | 194 00 | | 222 00 | |
| 139 43 | RCL | 167 00 | | 195 00 | | 223 00 | |

*Denotes 2nd function key

## CHI-SQUARE DISTRIBUTION

The chi-square density function is calculated by

$$p_{\nu}f(x) = \frac{e^{\frac{x}{2}}\, x^{\left(\frac{\nu}{2}-1\right)}}{2^{\frac{\nu}{2}}\,\Gamma\left(\frac{\nu}{2}\right)}$$

where $\nu$ = degrees of freedom, $0 < \nu \leq 141$, $x > 0$ and $\ln x \leq \frac{200 \ln 10}{(\nu - 2)}$

The following series expansion calculates the cumulative distribution:

$$P(x|\nu) = \left(\frac{x}{2}\right)^{\frac{\nu}{2}} \frac{e^{-\frac{x}{2}}}{\Gamma\left(\frac{\nu+2}{2}\right)} \left[1 + \sum_{r=1}^{\infty} \frac{x^r}{(\nu+2)(\nu+4)\ldots(\nu+2r)}\right]$$

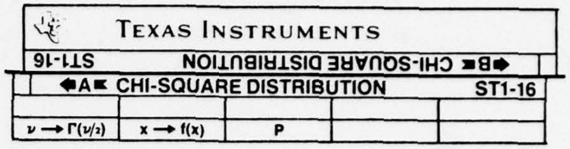
Where: $\Gamma(\nu+1) = \nu\Gamma(\nu) = \nu! = \nu(\nu-1)!$

and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

NOTES: 1. If the degrees of freedom is entered as a non-integer, it is rounded up to an integer for the calculation.
2. If $\nu$ or x is negative or zero, a flashing 0 is returned.
3. A $\nu$ must be entered before an x, and both must be entered before P is calculated.
4. A new x may be entered without changing $\nu$.
5. An x must be entered each time P is to be calculated.

Example: For $\nu = 10$, $\Gamma\left(\frac{\nu}{2}\right) = 24$ and if x = 8.5, f(x) = .0969533757, and P = .4198816863

Reference: *An Introduction to Probability and Stochastic Processes*; James L. Melsa, Andrew P. Sage; Prentice-Hall Electrical Eng. Series, 1973



TEXAS INSTRUMENTS

ST1-16 ◆B■ CHI-SQUARE DISTRIBUTION

◆A■ CHI-SQUARE DISTRIBUTION ST1-16

| | | | | |
|---|---|---|---|---|
| $\nu \rightarrow \Gamma(\nu/2)$ | $x \rightarrow f(x)$ | P | | |

### USER INSTRUCTIONS

| STEP | PROCEDURE | ENTER | PRESS | | | DISPLAY |
|---|---|---|---|---|---|---|
| 1 | Enter Program (A and B) | | | | | |
| 2 | Enter degrees | | | | | |
| | of freedom | $\nu$ | A | | | $\Gamma(\frac{\nu}{2})$ |
| 3 | Enter x | x | B | | | f(x) |
| 4 | Calculate P | | C | | | P |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 000 46 | *LBL | 028 75 | – | 056 04 | 4 | 084 52 | EE |
| 001 43 | RCL | 029 48 | *EXC | 057 49 | *PROD | 085 57 | *fix |
| 002 02 | 2 | 030 00 | 0 | 058 00 | 0 | 086 00 | 0 |
| 003 44 | SUM | 031 06 | 6 | 059 03 | 3 | 087 52 | EE |
| 004 00 | 0 | 032 95 | = | 060 46 | *LBL | 088 22 | INV |
| 005 08 | 8 | 033 22 | INV | 061 42 | STO | 089 52 | EE |
| 006 43 | RCL | 034 90 | *if zro | 062 43 | RCL | 090 57 | *fix |
| 007 00 | 0 | 035 43 | RCL | 063 00 | 0 | 091 09 | 9 |
| 008 02 | 2 | 036 43 | RCL | 064 04 | 4 | 092 22 | INV |
| 009 55 | ÷ | 037 00 | 0 | 065 75 | – | 093 80 | *if pos |
| 010 43 | RCL | 038 06 | 6 | 066 93 | • | 094 10 | *E' |
| 011 00 | 0 | 039 49 | *PROD | 067 05 | 5 | 095 90 | *if zro |
| 012 08 | 8 | 040 00 | 0 | 068 95 | = | 096 10 | *E' |
| 013 65 | X | 041 05 | 5 | 069 22 | INV | 097 42 | STO |
| 014 43 | RCL | 042 43 | RCL | 070 90 | *if zro | 098 00 | 0 |
| 015 00 | 0 | 043 00 | 0 | 071 44 | SUM | 099 07 | 7 |
| 016 04 | 4 | 044 05 | 5 | 072 59 | *$\pi$ | 100 55 | ÷ |
| 017 85 | + | 045 56 | *rtn | 073 30 | *$\sqrt{x}$ | 101 01 | 1 |
| 018 42 | STO | 046 46 | *LBL | 074 49 | *PROD | 102 42 | STO |
| 019 00 | 0 | 047 44 | SUM | 075 00 | 0 | 103 00 | 0 |
| 020 04 | 4 | 048 43 | RCL | 076 03 | 3 | 104 03 | 3 |
| 021 43 | RCL | 049 00 | 0 | 077 43 | RCL | 105 02 | 2 |
| 022 00 | 0 | 050 04 | 4 | 078 00 | 0 | 106 95 | = |
| 023 06 | 6 | 051 75 | – | 079 03 | 3 | 107 42 | STO |
| 024 95 | = | 052 01 | 1 | 080 56 | *rtn | 108 00 | 0 |
| 025 52 | EE | 053 95 | = | 081 46 | *LBL | 109 01 | 1 |
| 026 22 | INV | 054 42 | STO | 082 11 | A | 110 42 | STO |
| 027 52 | EE | 055 00 | 0 | 083 22 | INV | 111 00 | 0 |

*Denotes 2nd function key

## REGISTERS

| | | | |
|---|---|---|---|
| 00 | 05 Used | 10 | 15 |
| 01 Used | 06 Used | 11 | 16 |
| 02 Used | 07 Used | 12 | 17 |
| 03 Used | 08 Used | 13 | 18 |
| 04 Used | 09 | 14 | 19 |

## FLAGS

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

## PROGRAM LISTING

| DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY | DISPLAY | KEY |
|---|---|---|---|---|---|---|---|
| 112 04 | 4 | 140 56 | *rtn | 168 94 | +/– | 196 43 | RCL |
| 113 75 | – | 141 46 | *LBL | 169 54 | ) | 197 00 | 0 |
| 114 53 | ( | 142 12 | B | 170 22 | INV | 198 01 | 1 |
| 115 57 | *fix | 143 22 | INV | 171 23 | lnx | 199 95 | = |
| 116 00 | 0 | 144 52 | EE | 172 55 | ÷ | 200 49 | *PROD |
| 117 75 | – | 145 22 | INV | 173 43 | RCL | 201 00 | 0 |
| 118 93 | • | 146 80 | *if pos | 174 00 | 0 | 202 05 | 5 |
| 119 05 | 5 | 147 10 | *E' | 175 03 | 3 | 203 43 | RCL |
| 120 54 | ) | 148 90 | *if zro | 176 55 | ÷ | 204 00 | 0 |
| 121 52 | EE | 149 10 | *E' | 177 02 | 2 | 205 07 | 7 |
| 122 22 | INV | 150 42 | STO | 178 45 | $y^x$ | 206 42 | STO |
| 123 52 | EE | 151 00 | 0 | 179 43 | RCL | 207 00 | 0 |
| 124 57 | *fix | 152 02 | 2 | 180 00 | 0 | 208 08 | 8 |
| 125 09 | 9 | 153 45 | $y^x$ | 181 01 | 1 | 209 01 | 1 |
| 126 95 | = | 154 53 | ( | 182 95 | = | 210 42 | STO |
| 127 22 | INV | 155 43 | RCL | 183 42 | STO | 211 00 | 0 |
| 128 90 | *if zro | 156 00 | 0 | 184 00 | 0 | 212 06 | 6 |
| 129 42 | STO | 157 01 | 1 | 185 04 | 4 | 213 42 | STO |
| 130 43 | RCL | 158 75 | – | 186 42 | STO | 214 00 | 0 |
| 131 00 | 0 | 159 01 | 1 | 187 00 | 0 | 215 04 | 4 |
| 132 01 | 1 | 160 95 | = | 188 05 | 5 | 216 41 | GTO |
| 133 75 | – | 161 65 | X | 189 56 | *rtn | 217 43 | RCL |
| 134 01 | 1 | 162 53 | ( | 190 46 | *LBL | 218 46 | *LBL |
| 135 95 | = | 163 43 | RCL | 191 13 | C | 219 10 | *E' |
| 136 29 | *x! | 164 00 | 0 | 192 43 | RCL | 220 00 | 0 |
| 137 42 | STO | 165 02 | 2 | 193 00 | 0 | 221 20 | *1/x |
| 138 00 | 0 | 166 55 | ÷ | 194 02 | 2 | 222 00 | 0 |
| 139 03 | 3 | 167 02 | 2 | 195 55 | ÷ | 223 56 | *rtn |

*Denotes 2nd function key

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